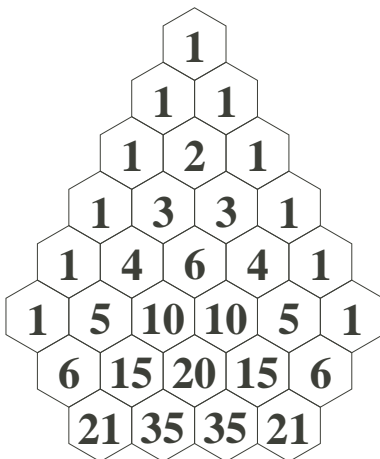
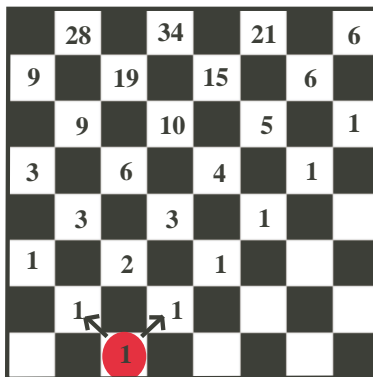


## Problem Set Solutions

1. Each cell in the pascal triangle actually gives us the number of possible paths there are that lead to the cell. We can look at the triangle and add the bottom row up to get  $21 + 35 + 35 + 21 = 112$  different paths.



2. Just like questions 1, we can use the concept of Pascal's triangle. However, since there are no extra paths to get to the edge of the board, those entries stay the same as the spot before it.  $28 + 34 + 21 + 6 = 89$ .

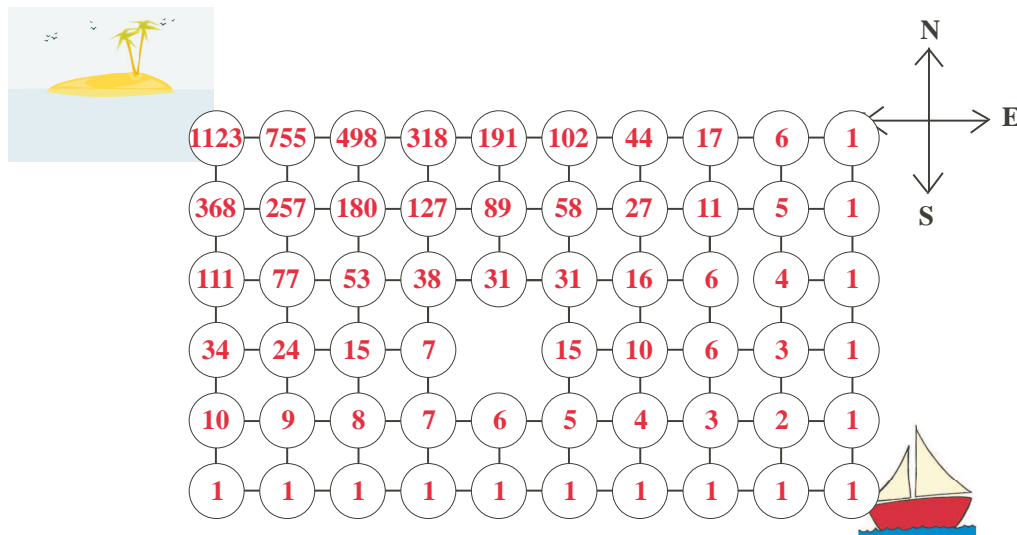


3. Filling in this portion of the triangle, we have

$$\begin{array}{r}
 1287 \quad \underline{1716} \quad \underline{1716} \quad 1287 \\
 3003 \quad 3432 \quad 3003 \\
 \underline{6435} \quad 6435 \\
 12870
 \end{array}$$

$$\therefore x = 1287, y = 3003, \text{ and } z = 12870$$

4. From the diagram, we can see that the number of paths is 1123.



5. The two shortest sticks are 1 and 2 units of length. The next shortest stick that would make it impossible to form a triangle is  $1 + 2 = 3$  units of length. The next length that would make it impossible to form a triangle would be  $2 + 3 = 5$ . Continuing this pattern, we have

$$3 + 5 = 8$$

$$5 + 8 = 13$$

$$8 + 13 = 21$$

$\therefore$  The shortest length of the longest stick is 21 units of length.

6.  $F(1) = 1, F(2) = 1 + 2, F(3) = 1 + 2 + 3$ . In general,

$$F(n) = 1 + 2 + 3 + \dots + n$$

$$2 \times F(n) = 2(1 + 2 + 3 + \dots + n)$$

$$= (1 + 2 + 3 + \dots + n) + (1 + 2 + 3 + \dots + n)$$

$$= [1 + n] + [2 + (n - 1)] + [3 + (n - 2)] \dots + [n + 1]$$

$$= [1 + n] + [1 + n] + \dots + [1 + n]$$

$$2 \times F(n) = n \times (1 + n)$$

$$F(n) = \frac{n(n + 1)}{2}$$

7. From question 6, we know that  $2 + 3 + \dots + 297 + 298 = F(298) - 1$

$$\begin{aligned} F(298) - 1 &= \frac{298(298 + 1)}{2} - 1 \\ &= \frac{298(299)}{2} - 1 \\ &= 44550 \end{aligned}$$

8. Let  $x$  be the square root of the number we're looking for. Then we have

$$x^2 + (x^2)^2 + x = x(x + x^3 + 1)$$

Breaking 276 into a product of its factors, we have

$$\begin{aligned} 276 &= 4 \times 69 \\ &= 4(4 + 64 + 1) \\ &= 4(4 + 4^3 + 1) \therefore x = 4 \end{aligned}$$

The number we are looking for is 16.

9. By making a table, we can see that there are 3 super primes less than 15

$n$	2	3	5	7	11	13
$2n - 1$	3	5	9	13	21	25

10. The three smallest primes are 2, 3, and 5, so the smallest perfect square with three distinct prime factors is

$$\begin{aligned} (2 \times 3 \times 5)^2 &= 30^2 \\ &= 900 \end{aligned}$$

11. The prime numbers between 12 and 25 are: 13, 17, 19, and 23. By multiplying any two of these numbers, we get

	13	17	19	23
13	N/A	221	247	299
17	221	N/A	323	391
19	247	323	N/A	437
23	299	391	437	N/A

Obviously, the numbers whose product is greater than 350 aren't the factors of the number we're looking for. For the numbers less than 300, we could see if multiplying by 2 could result in a number between 300 and 350. None of the numbers work, so the only answer is 323, whose factors are 17 and 19.

12. To maximize the number of cages, we need to minimize the number of birds in each cage. The first cage can contain one bird, and the second cage two birds, etc. until the 12<sup>th</sup> cage. by the 12<sup>th</sup> cage, we would have  $1 + 2 + 3 + \dots + 12 = 78$ . If we put 13 birds in the next cage, there will only be 9 birds left, which would be impossible not to repeat numbers. Thus,  $13 + 9 = 22$  need to go into the 13<sup>th</sup> cage. The maximum number of cages is 13.
13. Let the number be  $ABC$ . B must be either 4 or 8 since it is 4 times of C. B must also be greater than 7, because A is 7 less than B. From these two clues, B must be 8, so the number is 182.

14. To get a 3-digit number, A and C could be 5 and 6, or 4 and 6. If the pair is 5 and 6, then B and D won't add to more than 7, so E and F are both 1, which breaks the rule. Therefore, A and C must be 4 and 6. B and D won't add up to more than 8, so E is 1 and F is 0. This leaves us with 2, 3, and 5, where one of these numbers is the sum of the other two. It is clear that  $2 + 3 = 5$ . The number we are looking for is 5.

$$\begin{array}{r}
 \boxed{A} \boxed{B} \\
 + \boxed{C} \boxed{D} \\
 \hline
 \boxed{E} \boxed{F} \boxed{?}
 \end{array}$$