Sequences and Series

Week two questions

1. Starting at 13 and counting by 8s, a student counts 13, 21, etc. What is the 15^{th} number the student says?

2. The numbers $2, 5, 8, 11, 14, \ldots$ (where each number is three greater than the one preceding it) are written in order in a book, one hundred numbers to a page, beginning on page one. The number 11 111 will be found on what page?

3. Five yeast cells were placed in a laboratory dish at 4 p.m. The number of yeast cells doubles in every 10 minute interval. Determine the number of yeast cells in the dish (a) after 1 hour, and (b) after 2 hours.

4. The sum of $200 - 199 + 198 - 197 + 196 + \cdots + 2 - 1$ is equal to what value?

5. The sum of the first n terms of a sequence is $n(n+1)(n+2)$.
   Determine the 10^{th} term of the sequence.

6. What is the smallest positive integer x for which the sum $x + 2x + 3x + 4x + \cdots + 100x$ is a perfect square?

7. The sequence $6, -9, x, y$ is such that the first three terms form an arithmetic sequence and the last three terms form a geometric sequence. Determine the values of x and y.

8. a) Find all geometric sequences such that the sum of the first two terms is 2, and the sum of the first three terms is 3.

   b) For each of the sequences determined in part (a), calculate the sum of all terms having value less than 1.

9. In a sequence of six numbers, the first number is 4 and the last number is 47. Each of the numbers after the second is equal to the sum of the previous two numbers. Determine the sum of the six numbers.

10. The sum of fifty consecutive even integers is 3250. Determine the largest of the fifty integers.

11. The numbers in the sequence $2, 7, 12, 17, 22, \ldots$ increase by fives.
    The numbers in the sequence $3, 10, 17, 24, 31, \ldots$ increase by sevens.
    The number 17 appears in both sequences. What is the next number which appears in both sequences?

12. What is the 1987^{th} term in the following sequence?

    $$-2, -1, 0, 1, 2, -2, -1, 0, 1, 2, -2, -1, 0, 1, 2, -2, -1, 0, 1, 2, \ldots$$

    What is the sum of the first 1987 terms of the sequence?