Math Contest Preparation II

Intermediate Math Circles

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Last Week - Multiple Choice Questions

"This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct."

- get the simple ones right
- the right answer is not a wrong answer
- the right answer is one of the answers
- earlier probably means easier

Solutions to the problem set are now online.

Warm-up Problem

Two players take turns playing a game using four pennies on the 1 by 18 grid shown below. On a turn, a penny may be moved any number of positions to the right but it may not land on or move past another penny.

The player to make the last legal move wins the game.

Which player has a winning strategy and what is the winning strategy?

A				B							C			D		

What if you Don't Have Anything to Choose From?

This Week's Themes and Topics

- problem-solving strategies
- mathematical tools and tricks
- contest-writing strategies
- full solution questions
- writing proofs
- attacking hard problems
- CEMC contest details

Acknowledgement

Most problems used in today's session originate from past CEMC contests many of which are available on the CEMC website at http://www.cemc.uwaterloo.ca.

Convincing A Marker You Know the Answer

1. Emily sets up a lemonade stand. She has set-up costs of \$12.00 and each cup of lemonade costs her \$0.15 to make. She sells each cup of lemonade for \$0.75. Why is it not possible for her to make a profit of exactly \$17.00?

2. If $a > 0$ and $b > 0$, a new operation ∇ is defned as follows: $a \nabla b = \frac{a+b}{1+ab}$. For some values of x and y, the value of $x \nabla y$ is equal to $\frac{x+y}{17}$. Determine all possible ordered pairs of positive integers x and y for which this is true.

Draw a Picture

Determine the area of the triangle with coordinates $(-5, 2)$, $(4, 1)$ and $(2, -3)$.

Find a Pattern and Algebra is Your Friend

1. The first three terms of a sequence are 1, 2, 3. From the fourth term onwards, each term is calculated from the previous three terms using the rule "Add the first two and subtract the third." This means that the sequence starts with $1, 2, 3, 0, 5, -2, 7$. What is the 2010th term in the sequence?

2. The average of three consecutive multiples of 3 is a. The average of four consecutive multiples of 4 is a + 27. The average of the smallest and largest of these seven integers is 42. Determine the value of a.

Work Backwards

1. 337 orange balls and 476 white balls are placed in a box. What will the colour of the last ball in the box after the following?
   - Two balls are randomly removed from the box.
   - If the balls are the same colour, an orange ball is put back in the box.
   - Otherwise, a white ball is put back in the box.
   - This continues until one ball is left in the box.
2. Starting with input (m, n), Machine A outputs (n, m).
   Starting with input (m, n), Machine B outputs $(m + 3n, n)$.
   Starting with input (m, n), Machine C outputs $(m - 2n, n)$.
   Natalie starts with $(0, 1)$ and inputs it into one of the machines. She takes the output and inputs it into any one of the machines. She continues this process. Can the machine ever output $(2011, 1000)$?

Attacking Harder Problems

1. Determine the sum of the number of digits in the number 2^{2010} plus the number of digits in the number 5^{2010}.

2. A square array of dots with 10 rows and 10 columns is given. Each dot is coloured either blue or red. Whenever two dots of the same colour are adjacent in the same row or column, they are joined by a line segment of the same colour as the dots. If they are adjacent but of different colours, they are then joined by a green line segment. In total, there are 52 red dots. There are 2 red dots at corners, with an additional 16 red dots on the edges of the array. The remainder of the red dots are inside the array. There are 98 green line segments. How many blue line segments are there?

CEMC Contests

- mostly curriculum-based
- focus on logical thinking and problem solving instead of content
- last problems test ingenuity and insight
- calculators are permitted on all CEMC Math contests
- diagrams are not drawn to scale
- record your personal information correctly
- past contests available online (including solutions!)
- recognition given to top performers

Have you ever considered participating in a **computing contest**?

PCF and FGH

Pascal, Cayley, Fermat

- 25 multiple choice questions (bubble your answers!)
- 1 hour
- $5 \times 10 + 6 \times 10 + 8 \times 5 = 150$ marks over Parts A, B and C
- no penalty for an incorrect answer
- unanswered question worth 2, to a maximum of 10 unanswered questions
- February 24, 2011

Fryer, Galois, Hypatia

- 4 full solution questions (several parts each)
- 75 minutes
- 10 marks per question
- marks awarded for presentation as well as final answer
- Wednesday, April 13, 2011

CIMC, CSMC and The Euclid

Canadian Intermediate and Senior Math Contests

- **new** in 2011
- CIMC mostly for students in Grades 9 and 10
- CSMC mostly for students in Grades 11 and 12
- 6 answer only questions worth 5 marks each
- 3 full solution questions worth 10 marks each
- 2 hours
- Tuesday, November 22, 2011

Euclid

- primarily intended for students in final year of high school
- used to help determine scholarships for UWaterloo admissions
- 10 questions; some answer only and some full solution
- Tuesday, April 12, 2011

Homework Problems

Have fun!

Solutions will be posted next week.

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