Answers to Lecture 1 Problems:

1. The primes from 197 to 250 are: 197, 199, 211, 223, 227, 229, 233, 239, 241

2. (a)
$$\begin{array}{lllll} \Omega(2)=1 & \Omega(3)=1 & \Omega(4)=2 & \Omega(5)=1 & \Omega(6)=2 \\ \Omega(7)=1 & \Omega(8)=3 & \Omega(9)=2 & \Omega(10)=2 & \Omega(11)=1 \\ \Omega(12)=3 & \Omega(13)=1 & \Omega(14)=2 & \Omega(15)=2 & \Omega(16)=4 \\ \Omega(17)=1 & \Omega(18)=3 & \Omega(19)=1 & \Omega(20)=3 & \end{array}$$

   (b) $\Omega(100)=4$, $\Omega(400)=6$, $\Omega(800)=7$

   (c) Let $m=p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_n^{\alpha_n}$ and $n=p_1^{\beta_1}p_2^{\beta_2}\cdots p_n^{\beta_n}$. Then $\Omega(m)=\alpha_1+\cdots+\alpha_n$ and $\Omega(n)=\beta_1+\cdots+\beta_n$. Also,
$$mn=p_1^{\alpha_1+\beta_1}p_2^{\alpha_2+\beta_2}\cdots p_n^{\alpha_n+\beta_n}$$
   so
$$\begin{aligned}\Omega(mn)&=(\alpha_1+\beta_1)+(\alpha_2+\beta_2)+\cdots+(\alpha_n+\beta_n)\\&=\alpha_1+\cdots+\alpha_n+\beta_1+\cdots+\beta_n\\&=\Omega(m)+\Omega(n)\end{aligned}$$
   A function with this property is said to be **completely additive**.

   (d)
$$\begin{aligned}\Omega(210000)&=\Omega(210\times 1000)=\Omega(210)+\Omega(1000)\\&=\big[\Omega(21)+\Omega(10)\big]+\big[\Omega(10)+\Omega(100)\big]=[2+2]+[2+4]=10\end{aligned}$$

   (e) $\Omega(20!)=\sum_{i=2}^{20}\Omega(i)=1+1+2+1+2+1+3+2+2+1+3+1+2+2+4+1+3+1+3=36$

3. The n consecutive numbers $(n+1)!+2,(n+1)!+3,\ldots,(n+1)!+(n+1)$ are all composite. Hence, these form a prime gap of at least n numbers.

4. $22=11+11=5+17=3+19$ is the smallest even number that can be written as a sum of two primes in three ways.

   $34=17+17=23+11=29+5=31+3$ is the smallest even number that can be written as a sum of two primes in four ways.

5. Pick any odd number x greater than 5. Then $x=y+3$ where y is an even number greater than 2, so if Goldbach's Conjecture is true, then y can be written as a sum of two primes, so x is written as a sum of three primes. Similarly, for any even number greater x than 5, we can write $x=y+2$.