Solutions

Problem 1

(a) $m_{PQ} = \dfrac{-3-1}{-4+12} = -\dfrac{1}{2}$

$m_{QR} = \dfrac{-8+3}{6+4} = -\dfrac{1}{2}$

Therefore the points are collinear since $m_{PQ} = m_{QR}$, and each of these line segments share the midpoint Q.

(b) $|PQ| = \sqrt{(-4+12)^2 + (-3-1)^2} = \sqrt{80} = 4\sqrt{5}$

$|QR| = \sqrt{(6+4)^2 + (-8+3)^2} = \sqrt{125} = 5\sqrt{5}$

$|PR| = \sqrt{(-12-6)^2 + (1+8)^2} = \sqrt{405} = 9\sqrt{5}$

By the triangle inequality $|PQ| + |QR| \leq |PR|$, and the equality is satisfied when the points are collinear.

Thus, the points are collinear if $|PQ| + |QR| = |PR|$:

$$\begin{aligned} |PQ| + |QR| &= 4\sqrt{5} + 5\sqrt{5} \\ &= 9\sqrt{5} \\ &= |PR| \end{aligned}$$

Problem 2

$$\begin{aligned} m_{TA} &= m_{AW} \\ \frac{y+2}{-2-0} &= \frac{0-y}{4+2} \\ 6(y+2) &= 2y \\ 4y &= -12 \\ y &= -3 \end{aligned}$$

Problem 3

$$\begin{aligned} m_{AB} &= -\frac{1}{m_{BC}} \qquad \text{since lines } AB \text{ and } BC \text{ are perpendicular} \\ \frac{b-0}{0+2} &= -\frac{8-0}{0-b} \\ \frac{b}{2} &= \frac{8}{b} \\ b^2 &= 16 \\ b &= \pm 4 \end{aligned}$$

Problem 4

Let $W = (x, 0)$

$$\begin{aligned} 13^2 &= (x-7)^2 + (0-5)^2 \\ 169 &= x^2 - 14x + 49 + 25 \\ 0 &= x^2 - 14x - 95 \\ 0 &= (x+5)(x-19) \\ x = -5 &\text{ or } x = 19 \end{aligned}$$

$\therefore$ the coordinates of W are $(-5, 0)$ or $(19, 0)$

Problem 5

Let $A = (a, 7a)$ and $B = (b, b)$ where $a > 0$ and $b > 0$. We have

$$\begin{aligned} OB &= OA \\ (OB)^2 &= (OA)^2 \\ b^2 + b^2 &= a^2 + (7a)^2 \\ 2b^2 &= 50a^2 \\ b^2 &= 25a^2 \\ b &= 5a \end{aligned}$$

The slope of AB is

$$\begin{aligned} m_{AB} &= \frac{7a-b}{a-b} \\ &= \frac{7a-5a}{a-5a} \\ &= \frac{2a}{-4a} \\ &= -\frac{1}{2} \end{aligned}$$

Problem 6

(a) The midpoint is

$$\begin{aligned} M &= \left(\frac{-2+10}{2}, \frac{-11+5}{2}\right) \\ &= \left(\frac{8}{2}, \frac{-6}{2}\right) \\ &= (4, -3) \end{aligned}$$

(b) The using Pythagorean Theorem

$$\begin{aligned} AM &= \sqrt{(-2-4)^2 + (-11-(-3))^2} \\ &= \sqrt{6^2+8^2} \\ &= \sqrt{36+64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

$$\begin{aligned} MB &= \sqrt{(10-4)^2 + (5-(-3))^2} \\ &= \sqrt{6^2+8^2} \\ &= 10 \end{aligned}$$

$$\begin{aligned} MC &= \sqrt{(12-4)^2 + (3-(-3))^2} \\ &= \sqrt{8^2+6^2} \\ &= 10 \end{aligned}$$

(c) The slope of AC is

$$\begin{aligned} m_{AC} &= \frac{-11-3}{-2-12} \\ &= \frac{-14}{-14} \\ &= 1 \end{aligned}$$

The slope of BC is

$$\begin{aligned} m_{BC} &= \frac{5-3}{10-2} \\ &= \frac{2}{-2} \\ &= -1 \end{aligned}$$

$m_{AC} \times m_{BC} = (1)(-1) = -1 \therefore \angle ACB = 90°$

Problem 7

If the ratio of the length of AQ to the length of QB is $3:5$, then Q is located $\frac{3}{8}$ of the way from A to B.

The x-coordinate for Q is

$$\begin{aligned} 2 + \frac{3}{8}(10-2) &= 2 + \frac{3}{8}(8) \\ &= 2+3 \\ &= 5 \end{aligned}$$

The y-coordinate for Q is

$$\begin{aligned}-4+\frac{3}{8}[8-(-4)] &= 2+\frac{3}{8}(12)\\ &= -4+\frac{9}{2}\\ &= \frac{1}{2}\\ &= 0.5\end{aligned}$$

$\therefore Q = (5, 0.5)$