



University of Waterloo
Faculty of Mathematics



Centre for Education in
Mathematics and Computing

Grade 7 & 8 Math Circles

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Commission, Taxes & Discounts

Salary: Commission

In order to motivate sales, companies pay employees by commission. This means that employees get paid a certain amount (usually a percentage) for each product they sell. The percentage at which the employee gets the commission is called the **commission rate**. The more products you sell, the more you get paid. People who get commission usually get an hourly rate as well (just in case they aren't able to sell anything).

Problem: Jonas sells water. He gets 15% commission on all his sales.

a) One day, he sold \$5000 worth of water. How much did he earn that day?

Jonas' commission is 15% of his sales, so when he sold \$5000 of water, he gets 15% percent of that \$5000.

$$\begin{aligned} 15\% \text{ of } \$5000 &= 0.15(\$5000) \\ &= \$750 \end{aligned}$$

\therefore Jonas earned \$750 that day.

b) Last week after the drought, he made \$4500 of commission. How much water (in dollar amounts) did he sell?

Let x represent the sales Jonas made last week. Jonas made \$4500 commission, so we have

$$\begin{aligned} 15\% \text{ of } x &= 4500 \\ 0.15x &= 4500 \\ x &= \frac{4500}{0.15} \\ x &= 30,000 \end{aligned}$$

\therefore Jonas sold \$30,000 worth of water last week.

Problem: Justine works in an antique store. She gets paid 5% of the of anything she sells in the store, as well as \$12 per hour. She came into the store at 10:00a.m and left for lunch at 1:00p.m. She came back at 2:00p.m.

- a) In the morning, she sold a marble dining table for \$8000, two combs for \$200 each and a vase for \$500. How much did she earn in the morning? (remember to include the hourly salary!)

Justine worked from 10:00a.m to 1:00p.m, which means she gets $\$12 \times 3 = \36 from her hourly rate. From her sales, we have

$$\text{commission: } (8000 + 2 \times 200 + 500)(0.05) = 445$$

$$\text{commission plus hourly salary: } 445 + 36 = 481$$

\therefore Justine earned \$481 in the morning.

- b) After lunch, Justine worked for 4 more hours and sold a sculpture for \$5000 and an oak desk for \$1000. What is the total amount that Justine earned so far today?

Combining the hourly rate and commission into one step, we have

$$481 + (12)(4) + (5000 + 1000)(0.05) = 829$$

\therefore Justine earned \$829 so far today.

- c) Justine plans to close at 7:00p.m, and wants to earn \$1500 in total for today. If her last customer came to buy a conference table, what price would help Justine reach her goal?

Justine came back from lunch at 2:00p.m and worked for 4 more hours, which means that it was 6:00p.m. Justine only has one more hour to work. Let x represent the price of the table.

$$1500 = 829 + (12)(1) + 0.05x$$

$$0.05x = 1500 - 829 - 12$$

$$x = \frac{659}{0.05}$$

$$x = 13,180$$

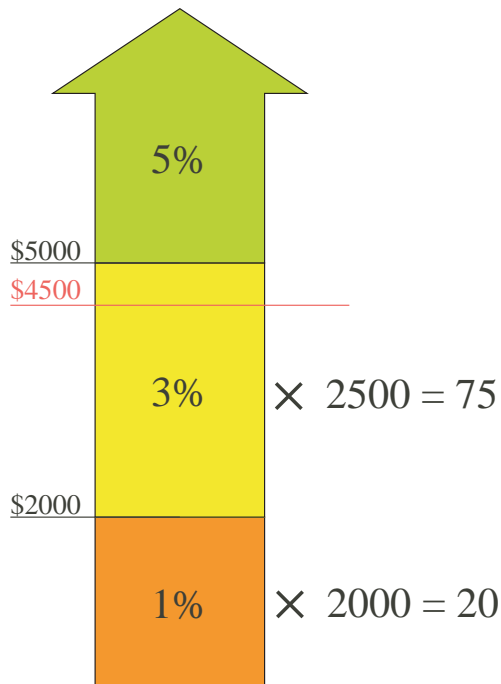
\therefore For Justine to reach her goal, the table should cost \$13,180.

Graduated Commission

To encourage employees even more to sell products, companies can introduce a **graduated commission**. If an employee sells more than a certain amount, the commission rate increases, so that the employee gets more pay per sale.

Problem: Desmond sells insurance. He gets 1% commission on the first \$2000 of his sales, 3% on the next \$3000 of his sales, and 5% for any more sales after that.

a) How much commission will he receive if he sells \$4500 worth of insurance?



Since Desmond sold more than the \$2000, the first commission level is “filled up”. That means there is $4500 - 2000 = \$2500$ remaining to fill up the second commission level.

$$(2000)(0.01) + (2500)(0.03) = 20 + 75 \\ = 95$$

\therefore Desmond will receive \$95 if he sells \$4500 worth of insurance.

b) How much commission will he receive if he sells \$6000 worth of insurance?



Since Desmond sold more than the \$5000, the first and second commission level are “filled up”. That means there is $6000 - 5000 = \$1000$ left for the remaining commission level.

$$20 + 90 + 50 = 160$$

\therefore Desmond will receive \$160 if he sells \$6000 worth of insurance.

Problem: Harry makes pots at a pottery factory. He gets paid \$12 an hour and \$4 per pot for the first 10 pots, \$5 per pot for the next 14 pots and \$6 per pot for the pots made after that. It takes Harry 15 minutes to make a pot.

- a) How much does Harry make during an 8 hour shift?

Harry gets $12 \times 8 = \$96$ from hourly salary. We know that he can make a pot in 15 minutes, so he can make 4 pots in an hour, which means $8 \times 4 = 32$ pots during his shift.

$$\text{commission: } (\$4/\text{pot})(10 \text{ pots}) + (\$5/\text{pot})(14 \text{ pots}) + (\$6/\text{pot})(8 \text{ pots}) = \$158$$

$$\text{commission plus salary: } 158 + 96 = 254$$

\therefore Harry earns \$254 during an 8 hour shift.

- b) Harry has switched to a 6 hour shift. If he wants to earn as much as he did during his 8 hour shift, how long should it take him to make a pot?

Let p represent the number of pots Harry has to make. We see that if Harry makes 24 pots in 6 hours, he will get $(4)(10) + (5)(14) + (12)(6) = \182 . His goal is to make \$254.

$$182 + 6p = 254$$

$$6p = 254 - 182$$

$$p = \frac{72}{6}$$

$$= 12$$

Harry needs to make $10 + 14 + 12 = 36$ pots in 6 hours, which means 6 pots in an hour. It should take him 10 minutes to make a pot.

- c) Harry's boss switched him back onto the 8 hour shift, and decided to make the commission rate the same no matter how many pots Harry makes. If Harry improved his speed to the rate of part b, what should the new commission rate be (per pot) so that Harry makes the same amount of money as he did before?

Let x represent the new commission rate. Harry can make 6 pots per hour, which means $8 \times 6 = 48$ pots

$$(8)(12) + 48x = 254$$

$$48x = 254 - 96$$

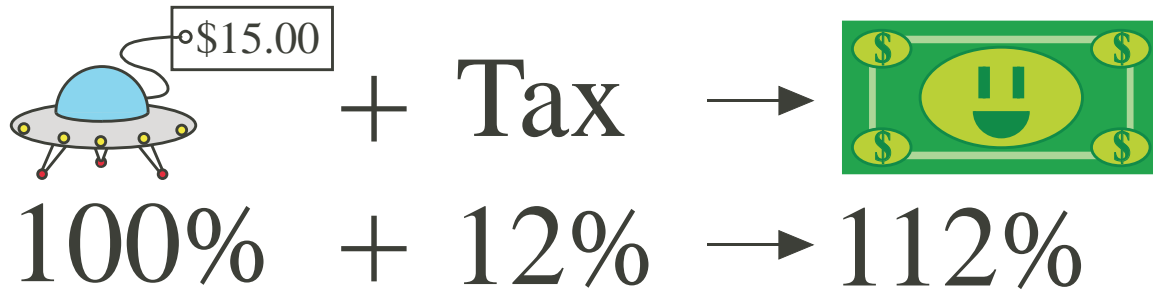
$$x = \frac{158}{48}$$

$$\approx 3.29$$

\therefore The new commission rate should be about \$3.29 per pot.

Tax

Governments charge taxes on the products we buy so that they can raise money to spend on things like roads, schools, hospitals and healthcare. With tax, we add on a percentage of what the item costs. Suppose we have a 12% tax rate and we want to buy a toy spaceship for \$15.



The tax, which is 12% of the price is added onto what we pay for the toy. In the end, you pay for 112% of the 15 dollars!

$$\begin{aligned} 112\% \text{ of } 15 &= 1.12 \times 15 \\ &= \$16.80 \end{aligned}$$

Problem: Max buys a bicycle for \$210 with 9% tax.

a) How much tax does Max pay?

$$\begin{aligned} \$210 \times 0.09 &= \$18.90 \\ \therefore \text{Max pays } &\$18.90 \text{ tax} \end{aligned}$$

b) How much does Max pay in total?

Let's use two different methods.

Method A $\$210 + \$18.90 = \$228.90$	Method B $\$210 \times 1.09 = \228.90
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Both methods work! Max pays \$288.90 in total.

c) Because the bicycle had a discount, Max only spent \$150. How much was the bike *before* tax?

Let x be the price before tax.

$$\begin{aligned} 1.09x &= 150 \\ x &= \frac{150}{1.09} && \therefore \text{The price before tax is } \$137.61 \\ x &= 137.61 \end{aligned}$$

Discounts

When new products come in, stores want to clear the older products. To make the older products more attractive, stores lower the price. The lowering of the price of products is called a **discount**, or **mark-down**. With discounts, a percentage is taken off. Suppose we have a 30% discount on a \$350 bowling ball.



Taking away 30% from 100%, we have 70% left. In the end, the bowling ball is 70% of the the 350 dollars!

$$\begin{aligned} 70\% \text{ of } 350 &= 0.70 \times 350 \\ &= \$245.00 \end{aligned}$$

Problem: Linh is buying a belt for \$30 with a 15% discount.

a) How much is discounted off the belt?

$$\begin{aligned} \$30 \times 0.15 &= \$4.50 \\ \therefore \$4.50 &\text{ is discounted off the belt.} \end{aligned}$$

b) What is the price of the belt after the discount?

Let's use two different methods.

Method A	Method B
$\$30 - \$4.50 = \$25.50$	$\$30 \times 0.85 = \25.50

\therefore The price of the belt after the discount is \$25.50.

c) After adding 15% tax, what percentage of the original price did Linh have to pay?

After the discount, the percent went from 100% to 85%. From that 85%, we have to apply 15% tax.

$$85\% \times 1.15 = 97.75\% \therefore \text{Linh had to pay } 97.75\% \text{ of the belt's original price.}$$

$$\text{We can double check: } (25.5)(1.15) = 29.325 \quad (30)(0.9775) = 29.325$$

Comparing Prices: system of linear equations

We all want to pay less for the same quality of work or get paid more the the same amount of effort, so what if you are given two options to choose from? For example, Kayla is offering you \$20 per hour to teach her skating while Jeanine is offering you \$18 per hour plus \$30 just for showing up! Since Kayla is offering a higher hourly rate, she would pay more if you taught her past a certain number of hours. But how many hours would you need to teach so that both of them would pay the same?

Let K be the amount that Kayla is paying you.

Let J be the amount that Jeanine is paying you.

Let t be the time that you spend on teaching.

We can create equations to relating the amount of time that you're teaching and the amount of money you are getting.

Kayla is paying \$20/hr, so we have

$$K = 20t \tag{1}$$

Jeanine is paying \$18/hr plus \$30, so we have

$$J = 18t + 30 \tag{2}$$

We want to see when Kayla and Jeanine would be paying the same amount. In other words, we want to find a time t such that K and J to be the same. We use the **method of substitution** to do so. Since we want K and J to be the same, we can set it as

$$K = J$$

Since $K = J$, we can substitute equation (2) into equation (1):

$$\begin{aligned} K &= 20t \\ 18t + 30 &= 20t \end{aligned}$$

Now all we have to do now is solve for t !

$$\begin{aligned} 18t + 30 &= 20t \\ 30 &= 20t - 18t \\ 30 &= 2t \\ \frac{30}{2} &= t \\ t &= 15 \end{aligned}$$

You would have to teach for 15 hours for Kayla and Jeanine to pay the same amount.

Problem Set:

1. Amzad buys an easy bake oven for \$30 (after tax). If he pays \$3 tax, what is the tax rate?
2. Sofia wants to lower the price of her handmade jewellery from \$19 to \$15. What is the discount rate she should advertise?
3. Tristan saw a pair of sunglasses for \$40 with a 30% tax! What is the price after tax?
4. Luke saw a pair of sandals for \$40 after a %30 discount! What was the original price?
5. Jill used to live in Nova Scotia where she sold dreamcatchers for \$11. She now lives in Ontario and wants to sell dreamcatchers. If the sales tax is 15% in Nova Scotia and 13% in Ontario, what price should she set the dreamcatchers at so that customers in Ontario pay the same as what the customers in Nova Scotia paid?
6. Cameron sells insurance. He makes \$12 per hour plus 9% commission of whatever he sells. Today, he worked seven hours and sold \$2500 worth of insurance. How much did he earn?
7. Trevor works at a toy car dealership. He is on a graduate commission, making 0% of the first \$150, 10% of the next \$3000 and 15% of all sales afterwards. How much must he sell in order to make \$1200?
8. From question 7, if Trevor switches to a single commission rate, what rate should he have so that he sells the same amount as he did to make \$1200?
9. You are given a choice between working for \$15 an hour **or** a 5% commission rate of your sales. How much do you have to sell in an hour so that both salary choices are just as good?
10. You are given a choice between working for either \$30 an hour plus \$150 for showing up **or** a 15% commission rate of your sales. You can sell \$300 worth of sales in an hour. When is either option equally as good?
11. Tim sells computers and gets paid on a graduated commission with three levels. He gets a base commission rate for the first \$1000 of sales. The second commission rate is 1% more than the base commission rate for the next \$5000. The last commission rate is 1% less than double the second commission rate, and it applies to all sales afterwards. This week, he made \$650 commission from \$12000 of sales. What are the commission rates?
12. How much does a discount rate have to be so that the total (after 15% tax) comes to half the original price?
13. Lily wants to buy a ratchet set. She sees a set for \$100 with 30% off. She decides to buy it. When Lily goes to the cashier, she finds out that there is a second discount off of the reduced price, and then an additional \$5 discount off of that. After a 15% tax, Lily pays \$54.62. What was the second discount rate?
14. An item is discounted three times: 30%,15% then 7%. What single discount rate would reduce the price by the same amount?

Answers:

$$1. \frac{\$3}{\$30 - \$3} = 11.11\%$$

$$2. \frac{\$19 - \$15}{\$15} = 21.05\%$$

$$3. (\$40)(1.3) = \$52$$

$$4. \frac{\$40}{0.7} = \$57.14$$

5. In Nova Scotia, the customers pay $(\$11)(1.15) = \12.65 . For the customers in Ontario to pay, $\$12.65$ Jill would have to sell the dreamcatchers for $\frac{\$12.65}{1.13} = \11.19 .

$$6. (12)(7) + (0.09)(2500) = 309 \therefore \text{Cameron earned } \$309.$$

7. Trevor makes $(\$3000)(0.1) = \300 on his second commission level and $\$0$ on his first commission level. He needs to make $\$1200 - \$300 = \$900$ on his third commission level. In order to do that, he must sell $\frac{\$900}{0.15} = \6000 on his third commission level. In total, he needs to earn $\$150 + \$3000 + \$6000 = \9150 .

$$8. \frac{\$1200}{\$9150} = 13.11\%$$

9. To make $\$15$ per hour from a 5% commission rate, you would need to sell $\frac{\$15}{0.05} = \300 of sales per hour.

10. The salary for option 1 can be represented by $S_1 = 30t + 150$ where t is the number of hours you worked and S_1 is your salary. Since you sell $\$300$ worth of sales in an hour, option 2 can be represented by $S_2 = (0.15)(300)t = 45t$. By substitution, we have

$$45t = 30t + 150$$

$$45t - 30t = 150 \quad \text{Either options are just as good if you work for 10 hours.}$$

$$15t = 150$$

$$t = 10$$

11. Let x represent the base commission rate. We have

$$1000x + 5000(x + 0.01) + (12000 - 1000 - 5000)(2(x + 0.01) - 0.01) = 650$$

$$1000x + 5000(x + 0.01) + 6000(2x + 0.01) = 650$$

$$1000x + 5000x + 50 + 12000x + 60 = 650$$

$$18000x + 110 = 650$$

$$18000x = 540$$

$$x = 0.03$$

\therefore The commission rates are 3%, 4% and 7%

12. Let d be the discount rate. We set the selling price at 100%. The customer would pay

$$\begin{aligned}(100\%)(1 - d)(1.15) &= 50\% \\ (1 - d)(1.15) &= 0.5 \\ 1 - d &= \frac{0.5}{1.15} \\ 1 - \frac{0.5}{1.15} &= d \\ d &= 56.52\%\end{aligned}$$

13. Let d represent the second discount rate.

$$\begin{aligned}[100(0.7)(1 - d) - 5](1.15) &= 54.62 \\ 70(1 - d) - 5 &= \frac{54.62}{1.15} \\ 1 - d &= \frac{47.50 + 5}{(70)} \\ 1 - \frac{52.50}{(70)} &= d \\ d &= 25\%\end{aligned}$$

\therefore The second discount rate is 25%

14. After all the discounts, a product's price goes from 100% to $(100\%)(0.7)(0.85)(0.93) = 55.34\%$. So we need to find a d such that

$$\begin{aligned}(100\%)(1 - d) &= 55.34\% \\ 1 - d &= 0.5534 \\ 1 - 0.5534 &= d \\ d &= 44.66\%\end{aligned}$$