Warmup

1. What is the total of a $60 pair of socks after 13% tax?
   \[100\% + 13\% = 113\% \text{ So the total is } 113\% \text{ of } \$60.\]
   \[(1.13)(\$60) = \$67.80\]
   \[\therefore \text{ The total is } \$67.80.\]

2. What is the price before 10% tax if the total comes to $121?
   Let \(x\) represent the original price. The total represents 110% of the original price.
   \[1.1x = 121\]
   \[x = \frac{121}{1.1}\]
   \[x = \$110\]
   \[\therefore \text{ The price before tax is } \$110.\]

3. What is the price of a $75 jack-o-lantern after a 25% discount?
   \[100\% - 25\% = 75\% \text{ So the answer is } 75\% \text{ of } \$75.\]
   \[(0.75)(\$75) = \$56.25\]
   \[\therefore \text{ The total is } \$56.25\]

4. What is the price before a 10% discount if the discounted price is $180?
   Similar to question 2, the discounted price is $180 is 90% of the original price.
   \[\frac{\$180}{0.9} = \$200\]
   \[\therefore \text{ The price before discount is } \$200.\]
**Markup**

**Q:** How do stores make money?

**A:** Most of the time when you buy something from a store, you are helping the store gain money. The key to this idea is that the stores buy products for a low price and sell them back out at a higher price. The price at which the stores buy a product is the **cost** of the product, and the amount in which they raise the price by is called the **markup**. The **markup rate** is the percentage comparing the markup over the cost. After the the markup, the product is at its **selling price**.

**Example:** S-Mart buys each of its popsicles for a cost of $2. It is then marked up 40¢ to a selling price of $2.40. The markup up rate is \( \frac{40\text{c}}{200\text{c}} = 20\% \).

**Problem:** Shehan buys comic books for $15 each then marks them up by 30% to resell. What are the markup and selling price?

**Method 1:** The markup is 30% of the $15

\[(0.3)(15) = 4.5\]

The markup is $4.50.

That makes the selling price $15 + $4.50 = $19.50

**Method 2:** The markup is 30% of the cost of the comic book, so the selling price is 100% + 30% = 130% of the cost.

\[(1.30)(15) = 19.50\]

The selling price of the comic book is $19.50

The markup is \( (0.3)(15) = 4.50 \)
**Exercise 1:** After a 10% markup, the selling price of a pair of earrings is $24.20. What was the original cost of the earrings?

Let $x$ represent the original cost of the earrings.

\[
110\% \text{ of } x = \$24.20
\]
\[
1.1x = \$24.20
\]
\[
x = \frac{\$24.20}{1.1}
\]
\[
x = \$22
\]

∴ The original cost of the earrings is $22

**Exercise 2:** Hossein owns a lemonade stand. He can make 7 glasses of lemonade with 12 lemons and 2 cups of sugar. He buys 24 lemons for $12 and 16 cups of sugar from $8. Since his sister is helping him, Hossein needs to pay 10¢ for each glass of lemonade he sells. If Hossein’s goal is to make a profit of 50¢ each glass, what is his markup rate and selling price?

Hossein buys 24 lemons for $12, so each lemon costs \(\frac{\$12}{24} = \$0.50\).

Similarly, since 16 cups of sugar cost him $8, each cup costs \(\frac{\$8}{16} = \$0.50\).

To make 7 glasses of lemonade, it will cost Hossein \(12(\$0.50) + 2(\$0.50) = \$6 + \$1 = \$7\)

For 1 glass of lemonade, it will cost Hossein \(\frac{\$7}{7} = \$1\).

Hossein needs to pay his sister $0.10 and plans to make a profit of $0.50 for each glass, so his markup should be \(\$0.10 + \$0.50 = \$0.60\).

The markup rate should be \(\frac{\$0.60}{\$1} = 0.60 = 60\%\).

The selling price is \(\$1 + \$0.60 = \$1.60\).
Markdown
When stores need to clear out older inventory, they often lower prices so that people would find it easier to buy. The stores would lower the price of the product to the sale price. The amount that the store lowers to get from the selling price to the sale price is called the markdown. Usually, stores would advertise markdowns as a percentage off of the selling price; this percentage is called the markdown rate. Even though the products are marked down, the store can still get a profit.

Example: S-Mart buys each of its popsicles for a cost of $2. It is then marked up 40¢ to a selling price of $2.40. No one wants to buy the popsicles, so it is marked down by 24¢ to a sale price of $2.16. The markdown rate is $\frac{24\text{¢}}{240\text{¢}} = 10\%$. Even though the popsicle has been marked down by 10%, it is still sold for 16¢ more than what the store paid to get it.

Problem: Charp Chuter sells sky diving gear. They bought a helmet at a cost of $50. And are selling it for $90. At a Christmas sale, the helmet was marked down by 20%. How much did the store make after covering the cost for the helmet?

The sale price of the helmet is $100\% - 20\% = 80\%$ of the selling price.

$(0.80)(90) = 72$

The cost of the helmet is $50, so after covering the cost, the store makes $72 - 50 = 22$. 
Exercise 3: Tuner Express sells tuners for $30. During a clearance sale, the tuners are sold for $24. What is the markdown rate?

The markdown of the tuner is $30 − $24 = $6. The markdown rate is the percentage of the markdown over the selling price.

\[
\frac{\text{markdown}}{\text{selling price}} = \frac{$6}{$30} = 0.2 = 20\%
\]

∴ the markdown rate is 20%

Exercise 4: After having little success with his lemonade sale, Hossein decided to mark the lemonade down by 30%. How much money will he lose/gain for each glass of lemonade?

The sale price of a glass of lemonade is $100% − 30% = 70% of the original selling price.

\[(0.7)(\$1.60) = \$1.12\]

Hossein has to cover the cost of the ingredients and pay his sister before he can pay himself. After that, he will get \(\$1.12 − \$1.00 − \$0.10 = \$0.02\)

Exercise 5: Instead of gaining 2 cents, what should the markdown rate be if he wanted to gain $0.15?

Hossein wants to gain $0.15, so the sale price should be $1.00 + $0.10 + $0.15 = $1.25

This would make the markdown $1.60 − $1.25 = $0.35.

\[
\frac{\text{markdown}}{\text{selling price}} = \frac{$0.35}{$1.60} = 0.2188 = 21.88\%
\]

∴ The markdown rate should be 21.88%.
**Problem Set**

1. Super Store buys a basketball for $7.00 and marks it up by 35%. Find the markup and the selling price.

2. Surplus Plus sells soap for $6. If the Cost of the soap is $4, how much is the markup and what is the markup rate?

3. Beau Mak marked up its lipstick by 60%. The markup is $9.00. What is the cost and selling price?

4. Tim’s Coffee Shop sells bottled water for $2.10. If the markup rate is 20%, what is the cost and markup?

5. Stellar Sellers marked down a $70 basket by 24%. What is the markdown and sale price?

6. The sale price of a bunny doll is $15 after a 30% markdown. What is the selling price and markdown?

7. Copy Cat marked down their clipboards by 60%. The markdown was $42. What is the selling price and sale price of the clipboards?

8. Sweet Peas bought peas for $7 and marked it up by 20%. When no one bought from Sweet Peas, they decided to mark the peas down by 20%. What was the sale price of the peas?

9. Which product has the highest markup rate: iPod Nano, bottled water, or a text message? Answer the questions to find out!
   
   (a) An iPod Nano costs $45 to make, but sells for $160. Find the markup rate.
   
   (b) Bottled water sells for around $1.75 and costs the companies 4¢ to make. Find the markup rate.
   
   (c) A text message costs around 15¢, but it costs the company 0.3¢ to process. Find the markup rate.
   
   (d) Which product has the highest markup rate? Were you surprised?

10. Manufacturers make products and sell them in bulk to wholesalers. Wholesalers then sell these products in smaller bundles to retailers (stores), which then sell them to consumers. Through each step of the process, a 20% markup and 10% tax is applied. What markdown rate should the retailer apply so that after the 10% tax, consumers pay as if they are buying from the manufacturer without any markups or taxes?
Answers:

1. Markup: \((0.35)(\$7.00) = \$2.45\); Selling price: \(\$7.00 + \$2.45 = \$9.45\)

2. Markup: \(\$6 - \$4 = \$2\); Markup rate: \(\frac{\$2}{\$4} = 50\%\)

3. Cost: \(\frac{\$9}{0.6} = \$15\); Selling price: \(\$9 + \$15 = \$24\)

4. Cost: \(\frac{\$2.10}{1.2} = \$1.75\); Markup: \(\$2.10 - \$1.75 = \$0.35\)

5. Markdown: \((0.24)(\$70) = \$16.80\); Sale price: \(\$70.00 - \$16.80 = \$53.20\)

6. Selling price: \(\frac{\$15}{0.7} = \$21.43\); Markdown: \(\$21.43 - \$15.00 = \$6.43\)

7. Selling price: \(\frac{\$42}{0.6} = \$70\); Sale price: \(\$70 - \$42 = \$28\)

8. \((\$7)(1.2)(0.8) = \$6.72\)

9. (a) \(\frac{\$160 - \$45}{\$45} = 255.55\%\)

   (b) \(\frac{\$1.75 - \$0.04}{\$0.04} = 4275\%\)

   (c) \(\frac{15\text{¢} - 0.3\text{¢}}{0.3\text{¢}} = 4900\%\)

   (d) Text messages have the highest markup rate.

10. Let’s set the manufacture cost at 100%.

    The wholesaler pays \((100\%)(1.2)(1.1) = 132\%\).

    The retailer pays \((132\%)(1.2)(1.1) = 174.25\%\).

    The consumer would normally pay \((174.25\%)(1.2)(1.1) = 229.9968\%\).

    To get back to the 100%, we need to reduce the selling price by \(229.9968\% - 100\% = 129.9968\%\)

    \(\therefore\) The markdown rate is \(\frac{129.9968}{229.9968} = 56.52\%\)
Just for Fun: A group of students was surveyed about their hobbies. With the help of the clues, complete the Venn diagram.

- Nobody dislikes all three hobbies
- 36 students like to go shopping
- Out of the students who don’t like to exercise, half of that group likes to cook
- Out of the students who like more than one hobby, two thirds of that group like shopping and exercising
- 26 students like to exercise
- There are twice as many students who only like to shop compared to the number of students who only like to cook
- 16% of the whole group of students like all three hobbies
- There are 44 students