

Solutions

Patterns

100. $1 + 6 = 7$; $7 + 6 = 13$; $13 + 6 = \underline{19}$

	picture number	number of gumballs	number of toothpicks
	1	3	3
	2	4	5
200.	3	5	7
	4	6	9
	5	7	11
	6	8	13

300. $1^3 = 1, 2^3 = 8, 3^3 = 27, 4^3 = 64, 5^3 = 125$

400. $1 \times 3 = 3, 3 - 1 = 2, 2 \times 3 = 6, 6 - 1 = 5, 5 \times 3 = 15, 15 - 1 = 14$

	picture number	number of tiles
	1	4
	2	8
500.	3	12
	4	16
	5	20

Mystery

100. $200 \times 0.15 = 30$

200. $\frac{1}{4} \times 100 + \frac{1}{10} \times 100 = 25 + 10 = \35

300. $2 \times 3 \times 4 = 24$

400. $180^\circ - 60^\circ - 50^\circ = 70^\circ$; $180^\circ - 120^\circ = 60^\circ$;
 $\Rightarrow x = 180^\circ - 60^\circ - 70^\circ = 50^\circ$

500. There are 9 distinct letters in 'probability': p, r, o, b, a, i, l, t, y $\Rightarrow P = \frac{9}{26}$

Algebra

100. $s_{\text{avg}} = \frac{120 + 80}{2} = 100 \text{ km/h}$

200. $E = F + V - 2 = 6 + 8 - 2 = 12$

300. $F = \frac{9}{5} \times 10 + 32 = 50^\circ F$

400. Let x be the smallest of the five consecutive numbers, then
 $\frac{x + (x + 1) + (x + 2) + (x + 3) + (x + 4) + (x + 5)}{5} = 21 \Rightarrow \frac{5x + 15}{5} = 21 \Rightarrow x = 18$

500. $r \times s + u \times r + t \times r = r \times (s + u + t)$ Thus, this value will be maximized when $r = 5$, and $s, t, u = 2, 3, 4$
 $\Rightarrow r \times s + u \times r + t \times r = 5 \times (2 + 3 + 4) = 45$

Logic

100. Mary is the fourth child
 200. C. If there are 1, 2, or 4 false statements then there is a contradiction



300.
 400. The stream causes both you and the bobber to move, so you can act as if there is no stream and so the bobber does not move, and you travel at 7 km/h. Therefore it will take you 2 hours to travel 14 km to reach the bobber.
 500. Let x be the number of quarters Abby has. Abby then has $23 - x$ nickles (since she has a total of 23 coins).

$$\begin{aligned} 0.25x + 0.05(23 - x) &= 4.55 \\ 0.2x &= 4.55 - 1.15 \\ 0.2x &= 3.40 \\ x &= 17 \end{aligned}$$

Abby has 17 quarters (and 6 nickles).

Gauss

100. There are 5 prime numbers from 3 to 13: 3, 5, 7, 11, 13 $\Rightarrow P = \frac{5}{11}$
 200. To make the smallest numbers in a 2-digit number, the tens digit must be the smallest, then the units digit will be the next smallest. Since we are summing two 2-digit numbers, one of the numbers will be $5x$ and the other needs to be $6x$. To get the units for each number we take the next lowest numbers, i.e. $57 + 68 = 125$
 300. $\angle P = \angle Q_{\text{right}} = \frac{180 - 80}{2} = 50^\circ$, also, $\angle R = \angle Q_{\text{left}} = \frac{80}{2} = 40 \Rightarrow \angle PQR = \angle P + \angle R = 90^\circ$
 400. Each point can be joined to 7 others $\Rightarrow 8 \times 7 = 56$ line segments, however we double counted all line segments \Rightarrow there are $56/2 = 28$ line segments.
 500. See solution online, Gauss 8 2010, #24
http://www.cemc.uwaterloo.ca/contests/past_contests/2010/2010GaussSolution.pdf