Problem Set I – Answers with Explanations

Math Contest Preparation I – Intermediate Math Circles

Acknowledgement: These problems are from the 2007 European Pink Kangaroo contest.

1. **A** Andy gives away two but gains 4 and ends up with 10, so he must have started with 8.

2. **A** The total number of dots on two dice is $2 \times (1 + 2 + 3 + 4 + 5 + 6) = 42$ and subtracting the visible dots $1 + 2 + 3 + 4 + 6 = 15$ leaves 27.

3. **C** The number of members in the three years' time will be $32 + 16 = 48$ then $48 + 24 = 72$ and finally $72 + 36 = 108$.

4. **D** The base JN of the new triangles is $\frac{3}{4} \times \frac{1}{2} \times 96 = 36$.

5. **E** The actual number of marbles doesn't matter (so long as the number is divisible by 9). Bag A is left with $\frac{1}{3}$ of its original contents, while Bag C now has $\frac{5}{3}$ so the ratio is 1:5.

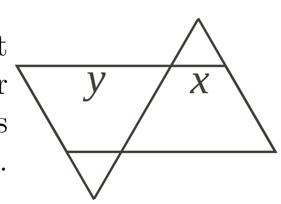
6. **A** The table can be filled in just by looking for a row or column with two identical entries already. Notice that the top row has two 0s so the missing entries are both 1. The completed table is

0	1	0	1
1	1	0	0
0	0	1	1
1	0	1	0

7. **C** The statements are mutually exclusive, and there are knights present so exactly one statement is true. Thus two groups must be liars, so there must be at least $2 + 4 = 6$ liars. This means that the first two groups must have been lying, so there are exactly $2 + 4 = 6$ liars.

8. **B** Considering both numbers as powers of 2, we find that $8^8 = (2^3)^8 = 2^{24}$ and $4^4 = (2^2)^4 = 2^8$. Now observe that $2^{24} = (2^8)^3$.

9. **B** Let y be the length of the triangle's edge and let x be the length that is cut off. Then the perimeter of the triangle is $3y$ and the perimeter of the parallelogram is $2(y + x) + 2(y - x) = 4y$. The difference is y which is 10 cm, so the perimeter of the original triangle is 30 cm.



10. **E** The sequence contains $20 \times 8 = 160$ letters, which we can number 1, 2, 3, etc. The first sweep leaves only the even numbers. The second sweep leaves only multiples of 4. The third sweep leaves only mulitples of 8, all of which are the letter O.

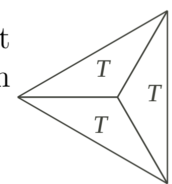
11. **D** There are 10 ways to pick a pair from five players, A, B, C, D, E: AB, AC, AD, AE, BC, BD, BE, CD, CE, DE. Each player appears in 4 pairs so he or she must play $4 \times 10 = 40$ games.

12. **C** Along the six edges the centre moves 1 cm (parallel to the edges). Around the six vertices it traces out an arc of radius $\frac{1}{2}$ cm and angle $60°$, which has length $\dfrac{60}{360} \times 2\pi \times \dfrac{1}{2} = \dfrac{\pi}{6}$.
The total distance $= 6 \times \left(1 + \dfrac{\pi}{6}\right) = 6 + \pi$.

13. **C** $P(A) = 1 \times \dfrac{2}{1} \times \dfrac{1}{10} = \dfrac{2}{110}$ $\qquad P(C) = 1 \times \dfrac{9}{11} \times \dfrac{6}{10} = \dfrac{54}{110}$

$P(B) = 1 \times \dfrac{8}{11} \times \dfrac{4}{10} = \dfrac{32}{110}$ $\qquad P(D) = 1 \times \dfrac{3}{11} \times \dfrac{2}{10} = \dfrac{6}{110}$

14. **E** By splitting area S into three small triangles T, we see that $S = 3T$; $L = 12T$; and $H = 6T$. Substituting these into the given expressions, we can see that only E is always true.



15. **D** If $10N$ is a square, then N must factorize as $10B^2$ for some integer B. Then $6N = 60B^2$ which factorizes as $60B^2 = 2^2 \times 3 \times 5 \times B^2$. We want $6N$ to be a cube, and to be minimal so B must contain 2, 3 and 5 as factors say $B = 30C$. Then $6N = 2^4 \times 3^3 \times 5^3 \times C^2$ and so $2C^2$ must be a cube. The smallest possible value for C is 2 so the smallest N is $2^5 \times 3^2 \times 5^3$. Now to find the number of factors of N, we choose from six powers of 2 (including $2^0 = 1$), three powers of 3 and four powers of 5; altogether this is $6 \times 3 \times 4 = 72$ choices.