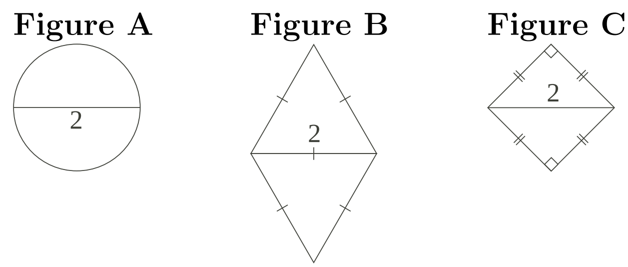
Problem Set II

Math Contest Preparation II – Intermediate Math Circles

Acknowledgement: These problems are taken from past CEMC contests.
Solutions: Full solutions for each question can be found online.
Note: You will probably find some of these problems quite challenging.

1. Of the three figures shown, which has the smallest area and which has the largest area? Explain how you determined your answer. *(#3b on 2008 Euclid Contest)*



2. Billy and Crystal each have a bag of 9 balls. The balls in each bag are numbered from 1 to 9. Billy and Crystal each remove one ball from their own bag. Let b be the sum of the numbers on the balls remaining in Billy's bag. Let c be the sum of the numbers on the balls remaining in Crystal's bag. Determine the probability that b and c differ by a multiple of 4. *(#7b on 2008 Euclid Contest)*

3. The first 30 positive integers can be written together in order, to form the 51-digit number:

$$x = 123456789101112131415161718192021222324252627282930$$

   (a) A positive integer that is the same when read forwards or backwards is called a *palindrome.* For example, 12321 and 1221 are both palindromes. Determine the smallest number of digits that must be removed from x so that the remaining digits can be arranged to form a palidrome. Justify why this is the minimum number of digits.

   (b) Determine the minimum number of digits that must be removed from x so that the remaining digits have a sum of 130. Justify why this is the minimum number of digits.

   (c) When the first 50 positive integers are written in order, the 91-digit number

$$y = 123456789101212...484950$$

   is formed. Determine the minimum number of digits that must be removed from y so that the remaining digits have a sum of 210 and can be arranged to form a palindrome. Justify your answer.

   (#4 on the 2008 Fryer Contest)

4. (a) Figure 1 shows a net that can be folded to create a rectangular box. Determine the volume and the surface area of the box.

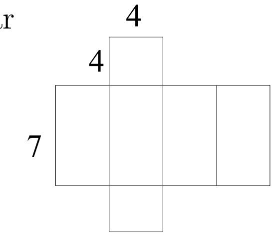


Figure 1

(b) In Figure 2, the rectangular box has dimensions 2 by 2 by 6. From point A, an ant walked to point B crossing all four of the side faces. The shortest path along which the ant could walk may be found by unfolding the box, as in Figure 3, and drawing a straight line from A to B. Determine the length of AB in Figure 3.

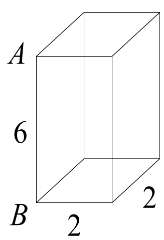


Figure 2

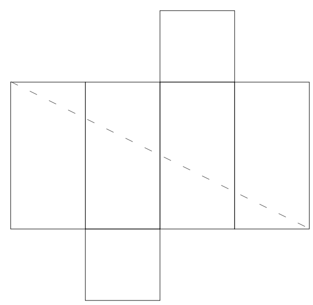


Figure 3

(c) In Figure 4, the rectangular block has dimensions 3 by 4 by 5. A caterpillar is at corner A. Determine, with justification, the shortest possible distance from A to G along the surface of the block.

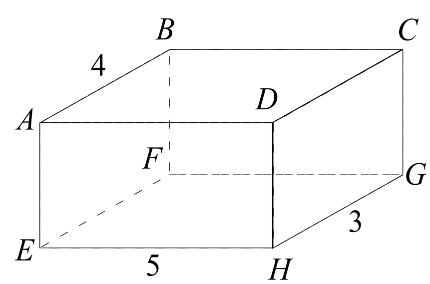


Figure 4

(#3 on the 2008 Fryer Contest)