



## Intermediate Math Circles

### Analytic Geometry II – Problems March 30, 2011

1. Prove, using analytic methods, that the diagonals of a parallelogram bisect each other.
2. For any quadrilateral  $ABCD$  with  $P$  the midpoint of  $AB$ ,  $Q$  the midpoint of  $BC$ ,  $R$  the midpoint of  $CD$ , and  $S$  the midpoint of  $DA$ , prove, using analytic methods, that  $PQRS$  is a parallelogram. (To help you get started, let  $A$  be at  $(0, 0)$ ,  $B$  be at  $(2b, 0)$ ,  $C$  be at  $(2c, 2e)$ , and  $D$  be at  $(2d, 2f)$ . This choice of points is helpful in finding midpoints later!)
3. Prove, using analytic methods, that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and one-half the length of the third side.
4. Determine the area of the triangle with vertices  $A(-2, -11)$ ,  $B(10, 5)$  and  $C(12, 3)$ .
5. Determine the area of the triangle with vertices  $P(-12, 1)$ ,  $Q(-4, -3)$  and  $R(6, -8)$ . Interpret your result.
6. A triangle with vertices  $X(4, 3)$ ,  $Y(6, p)$  and  $Z(0, 6)$  has area 7 units<sup>2</sup>. Determine all possible values for  $p$ .
7. Two telephone poles are  $p$  units apart. One pole is 8 units tall and the other is 10 units tall. A wire is strung from the top of each pole to the bottom of the other pole. The wires cross somewhere between the two poles. How high above the ground do the wires meet? Compare this answer to the similar example done during the Math Circles.
8. Determine the distance from the point  $Q(-3, 5)$  to the line  $2x - 7y + 1 = 0$ .
9. Calculate the coordinates of the foot of the perpendicular from the point  $(2, -6)$  to the line  $x - 3y - 2 = 0$ .
10. In  $\triangle ABC$ , with vertices  $A(6, 5)$ ,  $B(1, 1)$  and  $C(8, 1)$ , an altitude is drawn from  $A$  touching  $BC$  at  $D$ . Determine the length of the altitude  $AD$ . Develop two different solutions.
11. A point  $P$  is chosen on the line  $y = 2x + 3$  and a point  $Q$  is chosen on  $y = -x + 2$ . If the midpoint  $M$  of the line segment  $PQ$  is  $(2, 5)$ , determine the coordinates of  $P$  and  $Q$ .

