



Intermediate Math Circles

November 02, 2011

Counting II

Problem Set

1. If 50 different students try out for a team of 30 players, in how many different ways can the coach choose the team?
2. How many groups can be formed from 8 men and 5 women if:
 - a.) the group must have exactly 2 women and 2 men?
 - b.) the group can be any size, but must have at least one member and an equal number of men and women?
3. How many groups containing seven different numbers can be formed by selecting the numbers from the set $\{1, 2, \dots, 20\}$ such that
 - a.) 19 is the largest number in the group?
 - b.) 9 is the middle number in the group?
 - c.) the difference between the largest and smallest number in the group is equal to 14?
4. With a standard deck of 52 cards, a subset of 5 cards is called a hand.
 - a.) How many hands are there?
 - b.) How many hands contain exactly one pair? (2 of a kind and 3 different cards)
 - c.) How many hands have 4 of a kind?
5. How many **permutations** of the numbers $1, 2, \dots, 20$ taken 7 at a time
 - a.) contain 3 odd and 4 even numbers?
 - b.) contain 2 single-digit numbers and 5 two-digit numbers?
 - c.) have the 7 numbers arranged in increasing order?
6. Evaluate the following **without** the use of a calculator.

$$\text{a) } \frac{\binom{7}{3}}{\binom{7}{4}} \quad \text{b) } \frac{\binom{12}{8}}{\binom{9}{4}} \quad \text{c) } \frac{\binom{n}{3}}{\binom{n}{2}}, n \geq 3.$$

Answers will be emailed to you by Friday. If you disagree with an answer, try again. Next Wednesday, difficulties can be taken up in class.

Full solutions will be posted on our website after next Wednesday's class.