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Grade 7 & 8 Math Circles
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Dimensional Analysis

What is Dimensional Anaylsis

Dimensional Analysis is a method of solving problems that manipulates formulas through the use of various equivalent measurements.

Consider This...

The family recipe has been past down from generation to generation. Unfortunately, all the quantities of ingredients are measured in imperial units rather than metric units. The secret is the density of the dough, which can be changed by adding certain amounts of water. Unfortunately, this density is also given in imperial units. If only there was a method to easily convert between units!

This may be an exageration, but in many cases the use of dimensional analysis can make problems similar to this (and harder problems as well) much easier to solve and understand.

Understanding the Method

I Knew That!

- When a certain quantity is either multiplied or divided by 1, the quantity does not change (eg. $4 \times 1 = 4$ and $4 \div 1 = 4$)
- 1 can be expressed in many various fractions, as long as the numerator and denominator are the same (eg. $\frac{4}{4} = 1$ and $\frac{\text{feet}}{\text{feet}} = 1$)
- If a quantity shows up on the numerator and denominator of a fraction (or a string of fractions multiplied together), then the quantity can be divided out (i.e. cancelled out)

  $(\frac{12}{6}) \times (\frac{6}{4}) = \frac{12}{4} = \frac{3\times 4}{4} = 3$
  We can first divide out the two 6's.
  We then see that 12 is a multiple of 4, so we are able to divide out the two 4's.

Dimensional analysis just takes these properties and puts them together to divide out the units that we are given so that we are left with the units that we want.

Steps for Dimensional Analysis

1. Read through the question given
2. Determine what you want to know for your final answer and **what units you need** in the final answer
3. Determine **what units or measurement equalities you already know** from the unit conversion table or the question
4. Write the **measurement equalities in fraction form**
5. Write down a **starting point**
6. Use the measurement equalities and multiplication to **divide out the units you don't want** and **keep the units you need in the final answer** starting with your starting point (*Note:* You can use the reciprocal of your fractions that contain the unit equalities, as long as the proper numbers stay with the proper units).
7. Make your concluding statement

Perfect Timing

You are going to camp in exactly one week. You are so excited, you want to know how many minutes until you are off to camp. How many minutes are in one week?

Following our *steps for dimensional analysis*:

1. We read our question
2. Our final answer will be in minutes.
3. We know that there are 60 minutes per 1 hour, 24 hours per 1 day and 7 days per 1 week.
4. Putting these known equalities in fraction form we get:

$$\frac{60 \text{ min}}{1 \text{ hr}} \qquad \frac{24 \text{ hr}}{1 \text{ day}} \qquad \frac{7 \text{ day}}{1 \text{ week}}$$

5. Looking at the question, we can see that we are given a starting point of 1 week (*Note:* In some cases, you are given more than one starting point and you can choose which one to use)
6. $1 \text{ week} \times \frac{7 \text{ day}}{1 \text{ week}} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hr}}$

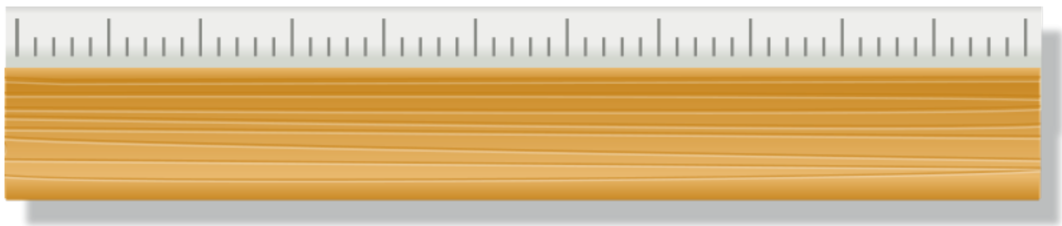
   We can now divide out units that are the same and appear in both the numerator and denominator to get:

   $1 \times \frac{7}{1} \times \frac{24}{1} \times \frac{60 \text{ min}}{1} = 10,080 \text{ min}$

7. Therefore, there are 10, 080 minutes in 1 week.

Unit Prefixes

Often, units have a prefix that proceeds the base unit. For example, centimetres have the prefix *centi* in front of metres. In fact, this same prefix can be used as a prefix for many different units including *centi*gram and *centi*litre.



In the prefix table, these prefixes are explained and each prefix is given a multiplication factor. This number explains how many of the base units are in these units. For example, the multiplication factor for centi is 10^{-2}, so we can deduce that in 1 cm there is 1×10^{-2} m$= 0.01$ m, in 2 cm there is 2×10^{-2} m$= 0.02$ m, and so on.

Example Set 1

1. How many metres (m) are in 1.579 gigametres(Gm)?
2. How many litres (L) are in 5 decilitres(dL)?
3. How many milligrams(mg) are in 1.5 grams(g)?

Unit Rate

A *unit rate* is a rate comparing to a single unit (i.e. there is a one in the denominator).

Example Set 2

1. Which of the following are unit rates?
   (a) 1 apple per day
   (b) $\frac{10 \text{ dm}}{100 \text{ cm}}$
   (c) 5 A's in every Report Card
   (d) $\frac{1 \text{ day}}{24 \text{ hr}}$

2. For each of the rates that were **not** unit rates in 1), convert them to a unit rate.

Area and Volume

Dimensional analysis can even be used to convert from one area unit or volume unit to another.

For example, if we would like to find out how many m^3 are in 24 cm^3, we would do the following:

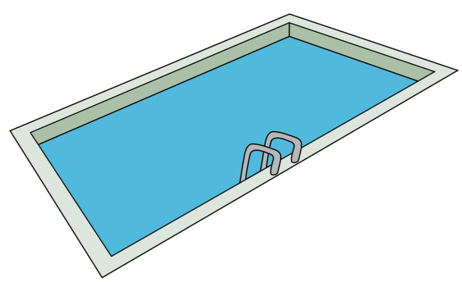
$$24 \text{ cm}^3 \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right) \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right) \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)$$

Notice how we find three cm units in the denominator and three cm units on top, so these will divide out, leaving us with:

$$24 \times \frac{1 \text{ m}}{100} \times \frac{1 \text{ m}}{100} \times \frac{1 \text{ m}}{100} = 0.000024 \text{ m}^3$$

Therefore there is 0.000024 m^3 in 24 cm^3.

Consider This...
Every year we have to fill up our pool that can hold 5 000 L of water. This year, I began to fill the pool at a rate of 25 L per minute for 20 minutes. After, the pool started leaking at a rate of 5 L per minute. How long did it take for me to fill up the entire pool?



Solution:

For the first 20 minutes:

$$\frac{25 \text{ L}}{\text{min}} \times 20 \text{ min} = 500 \text{ L}$$

Afterwards, the rate goes down to 25 L/min- 5 L/min=20 L/min to fill the remaining 5 000L-500 L=4 500 L:

$$\frac{1 \text{ min}}{20 \text{ L}} \times 4\ 500 \text{ L} = 225 \text{ min}$$

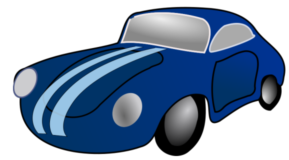
$$225 \text{ min} + 20 \text{ min} = 245 \text{ min}$$

Therefore, it takes 245 min to fill the entire pool (this is also 4 hours and 5 minutes).

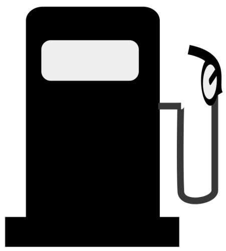
Example Set 3

For each of the following problems, round your final answers to two decimal places.

1. If you are driving your car at a speed of 95 kilometres per hour, how many metres are you going per minute?



2. You get lost in the middle of nowhere. Your car can hold 15.5 gallons of gas. At the moment, the gas gage indicates that you your tank is $\frac{1}{4}$ full.



If your car can drive 10.6 km per litre, and the next gas station is 175 km away. Will you make it to the gas station before running out of gas?

3. Susan decides to throw a surprise birthday party for her friend. She has \$65.00 to pay for the party. She invites 14 people to the party. She is hoping to have pizza and cake to eat and have extra money to decorate. Each person is expected to eat 4 pieces of pizza, each pizza has 8 pieces, and each pizza costs \$5.95. The cake will also cost \$15.00. If pizzas are able to be split up to the exact amount needed, how much money will Susan have left to decorate?

4. A canoe is going down a river at a speed of 10 km/hr **with** a current that has a speed of 100 km/hr.

If the destination is 20 miles away, how many minutes will it take the canoe to get to the destination?

5. A mixture contains 30% water. If you are given 1 quart of this mixture and know that the density is 3 grams per millilitre, how many grams **of water** do you have?

Unit Conversion Tables

Mass

16 ounces	1 pound
2, 000 pounds	1 ton
1 Newton	100 grams
1 pound	454 grams

Time

1 minute	60 seconds
1 day	24 hours
7 days	1 week
12 months	1 year
60 minutes	1 hour

Length

12 inches	1 foot
3 feet	1 yard
5,280 feet	1 mile
0.3048 metres	1 foot
0.9144 metres	1 yard

Volume

1 Litre	1.06 quarts
1 gallon	3.78 litres
4 quarts	1 gallon

Prefixes

Prefix	**Abbreviation**	**Multiplication Factor**
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
kilo	k	10^{3}
deci	d	10^{-1}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}