University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 7 & 8 Math Circles
November 9, 2011
Logic Puzzles

Solutions

Example Set 1

1. (a) The most that can be carried into the ten thousands column is 1 (since the largest digits for E and H is 9 and 8 and the largest carry over to hundreds is 1 and $9+8+1 = 18 < 20$). This means that $A = 1$ and since $T + 1 \geq 10 \Rightarrow T = 9$.

$$\begin{array}{r} \text{E19} \\ +\ \text{9H19} \\ \hline \text{1PPLE} \end{array}$$

From the ones column, $E = 8$ and a 1 is carried into the tens column implying $L = 1 + 1 + 1 = 3$.

$$\begin{array}{r} \text{819} \\ +\ \text{9H19} \\ \hline \text{1PP38} \end{array}$$

Since a 1 is carried into the thousands column, $9 + 1 = 10 + P \Rightarrow P = 0$. This means that $8 + H = 10 \Rightarrow H = 2$.

$$\begin{array}{r} \text{819} \\ +\ \text{9219} \\ \hline \text{10038} \end{array}$$

(b) The most that can be carried into the hundred thousands column is $1 \Rightarrow T = 1$.

$$\begin{array}{r} \text{SEVEN} \\ \text{1WO} \\ +\ \text{1HREE} \\ \hline \text{1WELVE} \end{array}$$

From the ten thousands column, it can see that either $S + 1 = 10 + W$ $\Rightarrow S = 9 + W$ or $1 + S + 1 = 10 + W \Rightarrow S = 8 + W$. Since $S \leq 9$, W must be either 0 or 1. But since every letter must be a different digit $\Rightarrow W = 0$.

$$\begin{array}{r} \text{SEVEN} \\ \text{10O} \\ +\ \text{1HREE} \\ \hline \text{10ELVE} \end{array}$$

Looking at the thousands column, we have 4 possible options:

$$\begin{aligned} E + H &= E \\ \Rightarrow H &= 0 \end{aligned} \qquad \begin{aligned} E + H &= 10 + E \\ \Rightarrow H &= 10 \end{aligned}$$

$$\begin{aligned} E + H + 1 &= E \\ \Rightarrow H &= 1 \end{aligned} \qquad \begin{aligned} E + H + 1 &= 10 + E \\ \Rightarrow H &= 9 \end{aligned}$$

The only H value that has not been used and is valid is $9 \Rightarrow H = 9$. Since the only options for S were 8 and $9 \Rightarrow S = 8$.

$$\begin{array}{r} \text{8EVEN} \\ \text{10O} \\ +\ \text{19REE} \\ \hline \text{10ELVE} \end{array}$$

From the ones digit, it can be seen that a 1 must be carried into the tens column and $N+O = 10$. For the tens column we either have $2E+1 = V$ or $2E + 1 = 10 + V$. This first option means E must be either 2 or 3, while the second option means E must be either 6 or 7.

Let $E = 7 \Rightarrow V = 4 + 1 = 5$ (from the tens column). From the hundreds column $1 + 5 + 1 + R = L \Rightarrow 7 + R = L$, but since all the valid digits above 7 have already been used, there is no way this can be true.

Let $E = 6 \Rightarrow V = 2 + 1 = 3$. We know that $N + O = 10$, but we have used all the digits that could be used to make combinations to 10, so this cannot be true.

Let $E = 3 \Rightarrow V = 6 + 1 = 7$. The only valid combination for N and O now is 4 and 6 (Let $N = 4$ and $O = 6$). The only digits that are allowed now are 2 and 5. From the hundreds column (and the fact that there is no carry-over into the hundreds column), $7+1+R = 10+L \Rightarrow R = 2+L$. Since 2 and 5 cannot fit into this expression, this cannot be true.

This leaves us with one more option for E. Let $E = 2 \Rightarrow V = 5$.

$$\begin{array}{r} \text{8252N} \\ \text{10O} \\ +\ \text{19R22} \\ \hline \text{102L52} \end{array}$$

From the hundreds column we have $5 + 1 + R = 10 + L \Rightarrow R = 4 + L$. From the digits that remain (3,4,6, and 7), the only possible pair that fit this description is 3 and 7. This means $R = 7$ and $L = 7$. Looking at the N and O value, we can see that it does not matter which one is 6 and which one is 4 (This means there are 2 possible answers).

$$\begin{array}{r} 82524 \\ 106 \\ +\ 19722 \\ \hline 102352 \end{array} \qquad \begin{array}{r} 82526 \\ 104 \\ +\ 19722 \\ \hline 102352 \end{array}$$

(c) Looking at the thousands digit, we can see that the largest value we can carry over into the next column is $1 \Rightarrow M = 1$.

$$\begin{array}{r} \text{SEND} \\ +\ \text{1ORE} \\ \hline \text{1ONEY} \end{array}$$

The only digits that S can stand for is 8 (if there is a carry over from the hundreds column) or 9. If $S = 8 \Rightarrow O = 0$, if $S = 9$ and there is a carry-over from hundreds column $\Rightarrow O = 1$ (*Note:* This cannot happen as $M = 1$), if $S = 9$ and there is no carry-over from the hundreds column $\Rightarrow O = 0$. In both the valid cases, $O = 0$.

$$\begin{array}{r} \text{SEND} \\ +\ \text{10RE} \\ \hline \text{10NEY} \end{array}$$

Looking at the hundreds column, we have $E = N$ or $E+1 = N$. Clearly, N and E cannot be the same digit so $E + 1 = N$. This also means that there is no carry over into the thousands column $\Rightarrow S = 9$.

$$\begin{array}{r} \text{9END} \\ +\ \text{10RE} \\ \hline \text{10NEY} \end{array}$$

From the tens column, either $N + R = 10 + E$ or $N + R + 1 = 10 + E$.

Assuming that $N + R = 10 + E$ is true, and substituting the fact that $E + 1 = N$:

$$\begin{aligned} N + R &= 10 + E \\ E + 1 + R &= 10 + E \\ R = 9 \end{aligned}$$

This cannot be true since $S = 9$.

The second statment $N + R + 1 = 10 + E$ must be true. Substituting the fact that $E + 1 = N$:

$$\begin{aligned} N + R + 1 &= 10 + E \\ E + 1 + R + 1 &= 10 + E \\ R &= 8 \end{aligned}$$

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  9END
+ 108E
 10NEY
```

Since the second statment was true, a 1 must be carried into the tens column meaning $D + E = 10 + Y$. Since Y cannot be 0 or 1 $D + E \geq 10 + 2 = 12$. With the digits we have left, the possible combinations whose sum is greater or equal to 12 are 5 and 7 or 6 and 7, meaning $D \in 5, 6, 7$ and $E \in 5, 6, 7$. This means Y is either 2 or 3.

Since $E + 1 = N$, we can now see that $E \in 5, 6$. This means that from the possible combinations above $D = 7$.

Let $E = 6 \Rightarrow 6 + 1 = N \Rightarrow N = 7$. But since $D = 7$ the assumption must be wrong.
This means that $E = 5$, meaning $N = 5 + 1 = 6$.

```
  955D
+ 1085
 1065Y
```

We know that $D + E = 10 + Y \Rightarrow D + 5 = 10 + Y \Rightarrow D = 5 + Y$ and we also know $Y \in 2, 3$. Since 5+3=8 and 8 has already been used, $Y = 2$ and $D = 5 + 2 = 7$.

```
  9557
+ 1085
 10652
```

2. From the ones column, $3E = 10x + 1$. The only number for E that allows this statement to be true is $E = 7$ (since $3(7) = 21$).

$$\begin{array}{r} \text{AB7} \\ \text{AC7} \\ +\ \text{AD7} \\ \hline 2011 \end{array}$$

There must be a 2 that is carried into the tens digit $\Rightarrow 2 + B + C + D = 10v + 1$. Now looking at the hundreds digit, we can see that $3A + v = 20$. The most v can be is 2 as $2 + B + C + D \leq 2 + 9 + 8 + 7 = 26$.

- If $v = 0$
  $3A + 0 = 20$
  There are no answers for A that is a valid digit
- If $v = 1$
  $3A + 1 = 20 \Rightarrow 3A = 19$
  There are no answers for A that is a valid digit
- If $v = 2$
  $3A + 2 = 20 \Rightarrow 3A = 18 \Rightarrow A = 6$

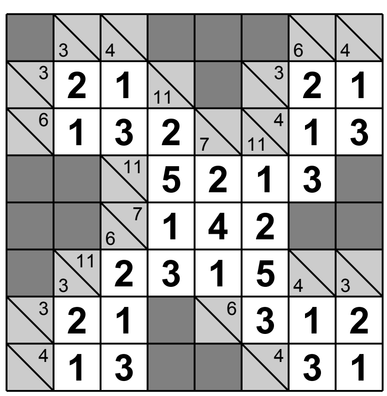
$$\begin{array}{r} \text{6B7} \\ \text{6C7} \\ +\ \text{6D7} \\ \hline 2011 \end{array}$$

It is known that $2 + B + C + D = 21 \Rightarrow B + C + D = 19$

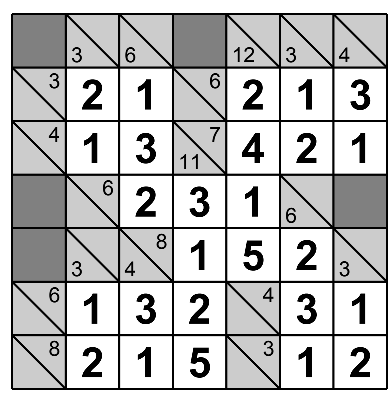
Therefore $A + B + C + D + E = 6 + 19 + 7 = 32$

Example Set 2

1.



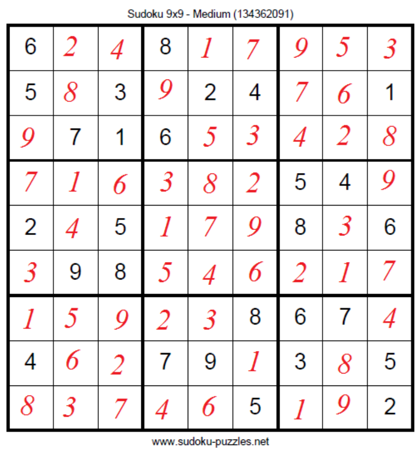
(a)



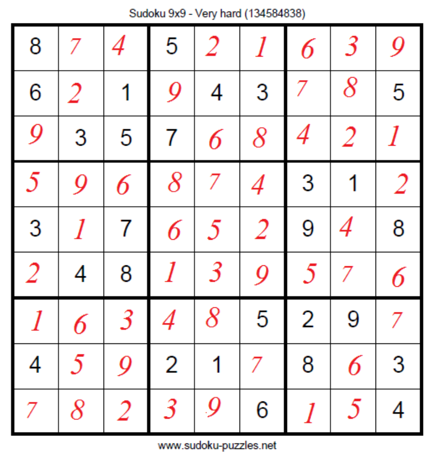
(b)

Example Set 3

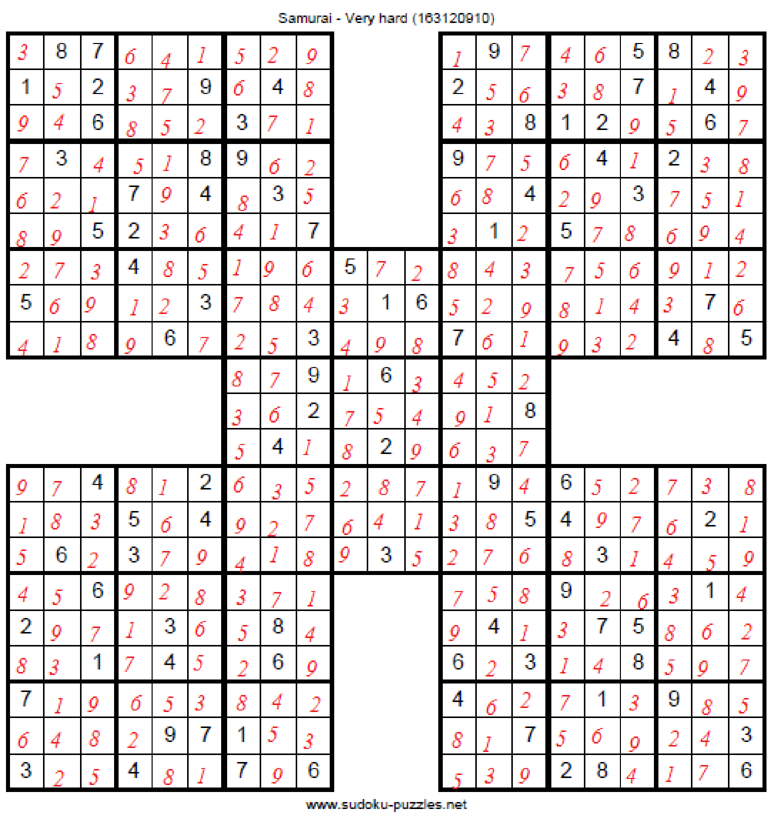
1.



(a)

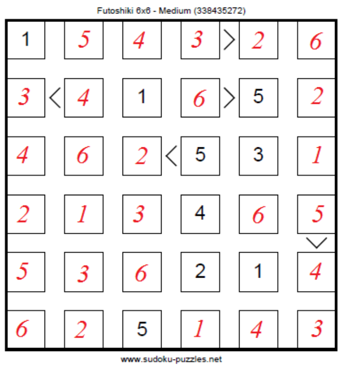


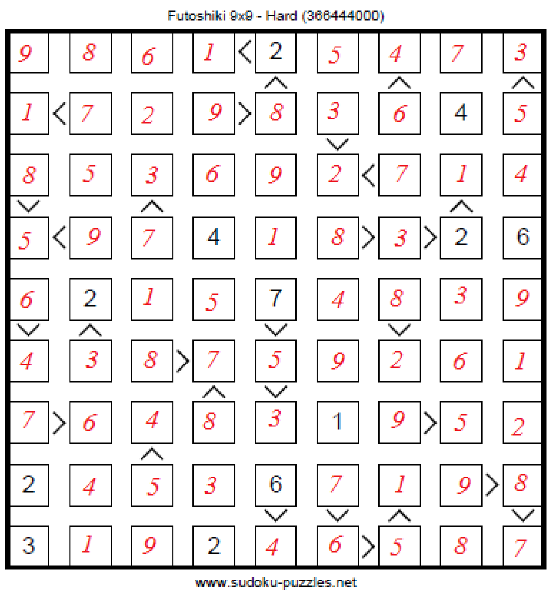
(b)

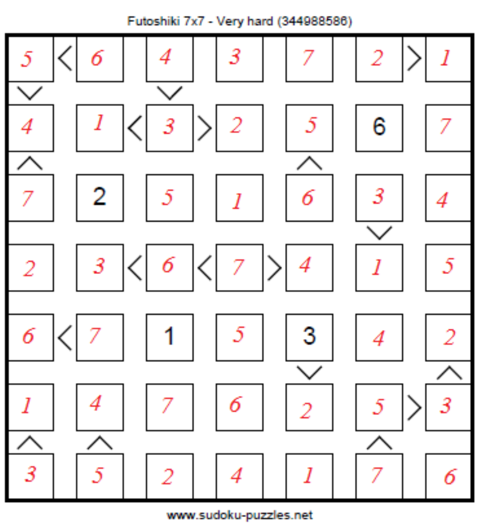


(c)

Example Set 4

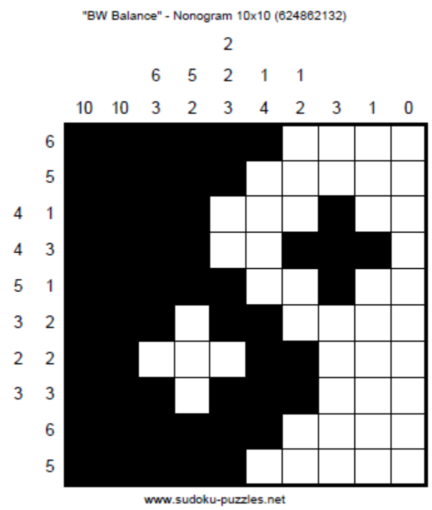
1. (a) 

(b) 

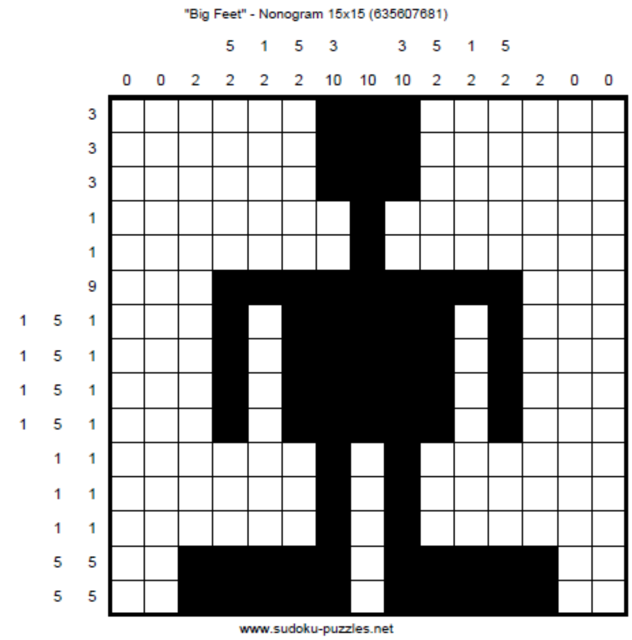
(c) 

Example Set 5

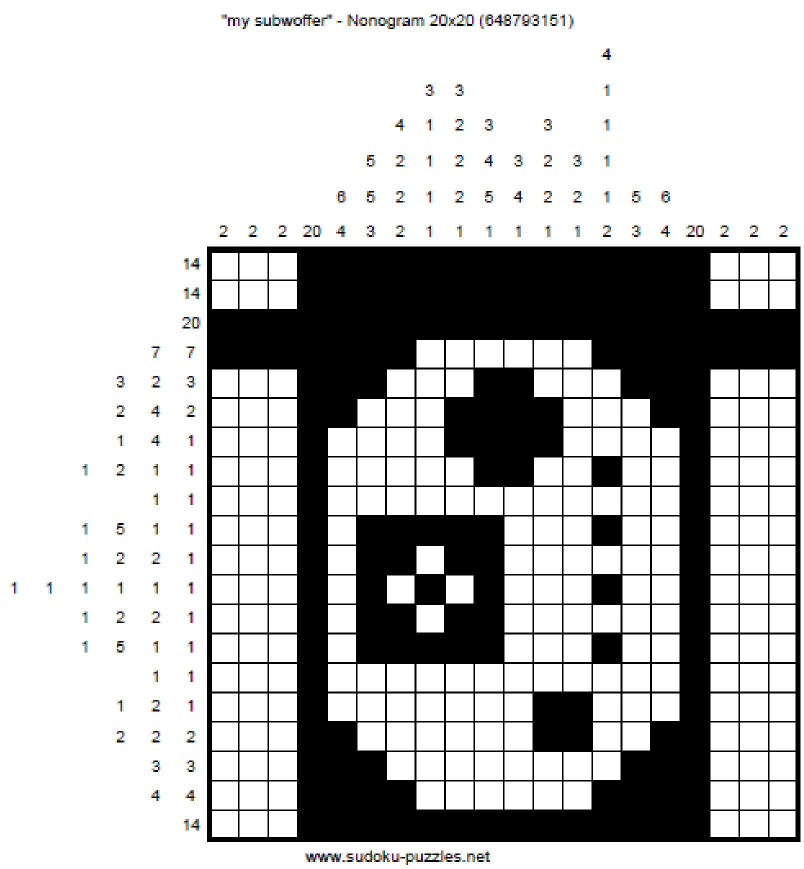
1.



(a)



(b)



(c)