

# Solutions

## Modular Arithmetic

### Question 1

*6 goes into 51 7 times with a remainder of 3.*

### Question 2

*When a number is divided by 6 the possible remainders are 0, 1, 2, 3, 4, and 5. Similarly when a number is divided by 4 the possible remainders are 0, 1, 2, and 3, and when dividing by 9 they are 0, 1, ... , 8.*

### Example 1

Odd	Even
1	2
3	4
5	6
7	8
9	10

The *even* numbers have remainder 0 when divided by 2, and the *odd* numbers have remainder 1 when divided by 2.

### Example 2

Remainder:

0	1	2
	1	2
3	4	5
6	7	8
9	10	

### Example 3

(a) 9 goes into 84 9 whole times with 3 left over, so  $84 \equiv 3 \pmod{9}$

(b)  $52 \equiv 2 \pmod{5}$

(c) How many times does 10 goes into -4?

Remember we are looking for some multiple of 10 that is **smaller** than -4, but as close to it as we can get.

This number is  $-10 = -1 \times 10$ .

-10 is 6 away from -4, therefore  $-4 \equiv 6 \pmod{10}$

### Example 4

1.  $3686132 \div 7 = 526590.285714\dots$

2.  $526590 \times 7 = 3686130$

3.  $3686132 - 3686130 = 2$ , so  $3686132 \equiv 2 \pmod{7}$

## Dates and Times

### Examples:

1. (a) There are 7 days in our week, so we will be working in mod 7.

7 goes into 165 23 times with 4 left over, or  $165 \equiv 4 \pmod{7}$ .

After 23 weeks it will be Wednesday again, and 4 days after that it will be Sunday.

(b)  $365 \equiv 1 \pmod{7}$ , Thursday.

(c)  $1000 \equiv 6 \pmod{7}$ , Tuesday.

2.  $21 \times 365 = 7665$

Leap years occurred in 2008, 2004, 2000, 1996, and 1992, so we add 5 more days to get total of 7670 days.

$7670 \equiv 5 \pmod{7}$ , and remember since we are looking into to past we are going **backwards** 5 days from Wednesday, therefore I was born on a Friday.

3. Rachel is  $13 \times 365 + 83 = 4828$  earth days old.

225 goes into 4828 21 times with 103 left over, so Rachel is 21 years and 103 days old on Venus. She will celebrate her 22<sup>nd</sup> Venusian birthday in  $225 - 103 = 122$  days.

## Cryptography

### Examples:

1. “M” is assigned to 12, and  $12 + 20 = 32 \equiv 6 \pmod{26}$ , so the code for “M” is “G”.

Continuing we get “GIXOFUL ULCNBGYNCW”.

2. “F” is assigned to 5, and  $5 + 26 - 16 = 15 \equiv 15 \pmod{26}$ , so “F” is decoded as “P”.

Continuing we get the original message “POLYGON”.

### Exercises:

1. Answers will vary.
2. Answers will vary.
3.  $8 \equiv 2 \pmod{3}$ , therefore it is 2:00.
4. We know that she is 21 Venusian years old, with 103 Earth days left over.  
 $103 \div 243 \approx 0.423868$ , so Rachel will celebrate her 22<sup>nd</sup> birthday in 0.423868 Venusian days.
5. Answers will vary.
6. Answers will vary.
7. “GEOMETRY”  
 We could also have done a one-step decryption with  $k = 41 \equiv 15 \pmod{26}$
8. No,  $53 \equiv 1 \pmod{3}$  so somebody has one more card than the other two people.
9. We are working with the modulus 360 because there are  $360^\circ$  in one rotation.  
 $1260 \equiv 180 \pmod{360}$  so Jon is now facing  $180^\circ$  clockwise from East, which is West.

10. We are looking for the smallest possible  $x$  greater than 10 such that  $x \equiv 2 \pmod{4}$  and  $x \equiv 1 \pmod{5}$ . Putting both congruence classes side by side we get:

$\equiv 2 \pmod{4}$	$\equiv 1 \pmod{5}$
10	11
14	16
18	21
22	<b>26</b>
<b>26</b>	
30	
34	

*Therefore the smallest possible number of loonies Philippa could have is 26.*