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Centre for Education in
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Grade 7 & 8 Math Circles
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Dimensional Analysis

Solutions

Example Set 1

1. $1.579 \text{ Gm} \times \frac{10^9 \text{ m}}{1 \text{ Gm}} = 1.579 \times 10^9 \text{ m}$
2. $5 \text{ dL} \times \frac{10^{-1} \text{ L}}{1 \text{ dL}} = 0.5 \text{ L}$
3. $1.5 \text{ g} \times \frac{1 \text{ mg}}{10^{-3} \text{ g}} = 1,500 \text{ mg}$

Example Set 2

1. (a), (c) and (d) are unit rates.
2. For (b) we have:

   $\frac{10 \text{ dm}}{100 \text{ cm}}$

   If we make it a unit rate for dm, we divide the top and the bottom by 10 to get:

   $\frac{1 \text{ dm}}{10 \text{ cm}}$

   If we make it a unit rate for cm, we divide the top and the bottom by 100 to get:

   $\frac{0.1 \text{ dm}}{1 \text{ cm}}$

   Both of these are unit rates and possible answers

Example Set 3

1. $\frac{95 \text{ km}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1000 \text{ m}}{1 \text{ km}} = \frac{95{,}000 \text{ m}}{60 \text{ min}} \approx \frac{1{,}583.33 \text{ m}}{\text{min}}$

   Therefore you are driving at a speed of approximately 1, 583.33 metres per minute

2. We first must calculate how much gas we have in our tank currently:
   15.5 gal$\times\frac{1}{4}$=3.875 gal

   We see that the mileage is given in kilometers per litre. Since we have nothing else in gallons, we will change gallons to litres:
   3.875 gal $\times \frac{3.78 \text{ L}}{1 \text{ gal}}$ =14.6475 L

   We now want our final answer to be in km, so we make the following conversion:
   14.6475 L $\times \frac{10.6 \text{ km}}{1 \text{ L}}$ =155.2635 km

   This is how far the car will make it with the current gas in the tank. We see that this is less than the next gas station.

   Therefore, you will not make it to the gas station before running out of gas.

3. 14 people $\times \frac{4 \text{ pieces}}{1 \text{ person}} \times \frac{1 \text{ pizza}}{8 \text{ pieces}} \times \frac{\$5.95}{1 \text{ pizza}}$

   $= 14 \times \frac{4}{1} \times \frac{1}{8} \times \frac{\$5.95}{1}$=\$41.65

   This is the price for the pizza. We must now add the price of the cake on top of that:
   \$41.65+\$15.00=\$56.65

   To determine how much Susan has left to decorate, we must find the difference between \$65.00 and the calculated sum above:
   \$65.00-\$56.65=\$8.35

   Therefore, Susan has \$8.35 left to decorate for the party.

4. It is important to first note that if the canoe is going down the river **with** the current, then the total speed will be the sum of the two individual speeds:
   10 km/hr+100 km/hr=110 km/hr

   We see now that our destination is given in miles, where the speed is in kilometres, so we will change miles to kilometers:
   $20 \text{ miles} \times \frac{5{,}280 \text{ ft}}{1 \text{ mile}} \times \frac{0.3048 \text{ m}}{1 \text{ ft}} \times \frac{1 \text{ km}}{1000 \text{ m}}$
   $= 20 \times \frac{5{,}280}{1} \times \frac{0.3048}{1} \times \frac{1 \text{ km}}{1000}$
   =32.18688 km

   Now taking our speed and the distance of the final destination, we can find how long (in hours) it will take to get to the destination
   $32.18688 \text{ km} \times \frac{1 \text{ hr}}{110 \text{ km}} = 32.18688 \times \frac{1 \text{ hr}}{110}$ =0.292608 hr

   Finally, we want our final answer to be in minutes, so we convert the number of hours we got above to minutes
   $0.292608 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}} = 0.292608 \times \frac{60 \text{ min}}{1}$ ≈17.56 min

   Therefore it will take approximately 17.56 minutes to get to the destination.

5. We first change the amount given from quarts to millilitres:
   $1 \text{ qt} \times \frac{1 \text{ L}}{1.06 \text{ qt}} \times \frac{1{,}000 \text{ mL}}{1 \text{ L}} = 1 \times \frac{1}{1.06} \times \frac{1{,}000 \text{ mL}}{1}$ =943.396 mL

   We now find how many grams of the **mixture** we have:
   $943.396 \text{ mL} \times \frac{3 \text{ g}}{1 \text{ mL}} = 943.396 \times \frac{3 \text{ g}}{1}$ =2, 830.188 g

   We now want to find how many grams of **water** we have. We know that 30% of the mixture is water, and we know the weight of the mixture so we must calculate:
   $30\% \times 2,830.188 \text{ g} = 0.30 \times 2,830.188 \text{ g} \approx$ 849.06 g

   Therefore you have approximately 849.06 grams of water.