

Math Circles 9 / 10

Preparing to Write Pascal, Cayley, Fermat

John Galbraith

Centre for Education in Mathematics and Computing
Faculty of Mathematics
University of Waterloo
Waterloo, Canada
www.cemc.uwaterloo.ca

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Agenda

- 1 Contest Information
- 2 Contest Format
- 3 Some General Comments About Preparing Through Examples
- 4 Working on Problems

Contest Information

Pascal for students in grade 9 or lower

Cayley for students in grade 10 or lower

Fermat for students in grade 11 or lower

Thursday, February 23, 2012

Registration Deadline has passed.

The Contest Format

From the instructions:

This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct.

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- 60 minutes
- 150 total marks
 - 10 questions in A part worth 5 marks each
 - 10 questions in B part worth 6 marks each
 - 5 questions in C part worth 8 marks each
- Each unanswered question is worth 2 marks each to a maximum of 10 unanswered questions.
- calculator permitted

The Contest: Should I Guess?

150 total marks (A) 10×5 (B) 10×6 (C) 5×8

Unanswered questions 2 marks each to a maximum of 10 blank

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Unanswered questions 2 marks each to a maximum of 10 blank

Score With **Random** Guessing:

Part A 10 right, Part B 8 right 2 wrong, Part C 2 right 3 wrong

$$\text{Score } 10 \times 5 + 8 \times 6 + 2 \times 8 = 114$$

The Contest: Should I Guess?

150 total marks (A) 10×5 (B) 10×6 (C) 5×8

Unanswered questions 2 marks each to a maximum of 10 blank

Score With **Random** Guessing:

Part A 10 right, Part B 8 right 2 wrong, Part C 2 right 3 wrong

$$\text{Score } 10 \times 5 + 8 \times 6 + 2 \times 8 = 114$$

Score With **No** Guessing:

Part A 10 right, Part B 8 right 2 blank, Part C 2 right 3 blank

$$\text{Score } 10 \times 5 + 8 \times 6 + 2 \times 8 + 5 \times 2 = 124$$

The Contest: Should I Guess?

150 total marks (A) 10×5 (B) 10×6 (C) 5×8

Unanswered questions 2 marks each to a maximum of 10 blank

Score With **Random** Guessing:

Part A 10 right, Part B 8 right 2 wrong, Part C 2 right 3 wrong

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Score With **No** Guessing:

Part A 10 right, Part B 8 right 2 blank, Part C 2 right 3 blank

$$\text{Score } 10 \times 5 + 8 \times 6 + 2 \times 8 + 5 \times 2 = 124$$

There is a difference between a random guess and an educated guess.

Contest Preparation

Most of the problems that we will look at have been taken from past Pascal and Cayley contests.

The key to success on any math contest is _____.

Writing a Math contest is different than solving math problems in general. The contest is timed! By doing many old contests you will pick up some tricks and techniques which will help you to write a good contest.

Go to http://www.cemc.uwaterloo.ca/contests/pcf_eWorkshop.html for the Pascal, Cayley, Fermat E-workshop.

• CEMC Home	Mathematics and Computing Contests				
• Mathematics and Computing Contests	Pascal, Cayley and Fermat eWorkshop				
• Registration	The Pascal, Cayley and Fermat eWorkshop consists of 4 quizzes designed to help students prepare for the Pascal, Cayley and Fermat contests.				
• Preparation					
• Results	Each quiz consists of 10 questions and is designed to be completed in 20 to 30 minutes. The topics and difficulty levels are designed to be a tool for interested students to prepare for these contests. The problems on each quiz generally progress from easier to harder.				
• Past Contests					
• Committees					
• Master of Mathematics for Teachers	Answers and hints for all quizzes within a given set are also provided.				
• Web Resources					
• Face-to-Face Workshops	Set 1	Pascal	Cayley	Fermat	Answers
• Order Books	Set 2	Pascal	Cayley	Fermat	Answers
• Sponsors and Donors	Set 3	Pascal	Cayley	Fermat	Answers
• About Us	Set 4	Pascal	Cayley	Fermat	Answers
• FAQ					

Contest Resources

You can also find old contests at:

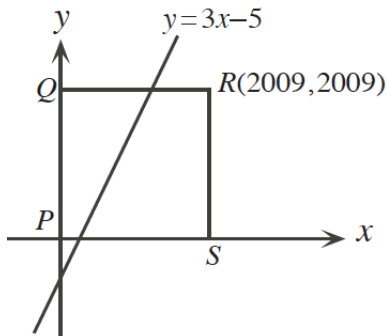
http://www.cemc.uwaterloo.ca/contests/past_contests.html#pcf

Pascal (Grade 9)											North and South America PDF
	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	
Contests	2001	2000	1999	1998	1997						North h PDF
	Download all as single PDF										
Solutions	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	
	2001	2000	1999	1998	1997						
Results	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	
	2001										
Cayley (Grade 10)											
	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	
Contests	2001	2000	1999	1998	1997						
	Download all as single PDF										
Solutions	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	
	2001	2000	1999	1998	1997						
Results	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	
	2001										
Fermat (Grade 11)											
	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	
Contests	2001	2000	1999	1998	1997						
	Download all as single PDF										
Solutions	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	
	2001	2000	1999	1998	1997						

WARM-UP #1

A *lattice point* is a point with integer coordinates. (For example, $(1, 4)$ is a lattice point but $(\frac{3}{2}, 4)$ is not.) The line $y = 3x - 5$ passes through the square $PQRS$ as shown in the diagram. If the coordinates of R are $(2009, 2009)$, then the number of lattice points on the line which are inside the square is

- (A) 666 (B) 667 (C) 668 (D) 669 (E) 670

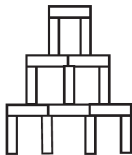


WARM-UP #2

From PCF EWorkshop Practice Pascal #1 Question 9

Dean is building a tower with blocks as shown in the diagram. The tower shown has three stories and uses 15 blocks. How many blocks are required for a tower of 80 stories?

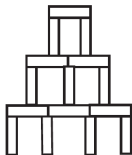
- (A) 400 (B) 6399 (C) 6496 (D) 6560 (E) 6723



WARM-UP #2 Solution 2

Dean is building a tower with blocks as shown in the diagram. The tower shown has three stories and uses 15 blocks. How many blocks are required for a tower of 80 stories?

- (A) 400 (B) 6399 (C) 6496 (D) 6560 (E) 6723



Problem Set #1

Work on these problems for the next 15 minutes. Try to work independently as much as possible. We will discuss the problems a little after the work time.

Problem Set #1 Answers

Answers to Problem Set 1:

1 D	2 A	3 D	4 B	5 D	6 D
7 E	8 D	9 E	10 B	11 E	12 D

Read Problems Carefully

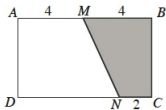
Were there any of the questions from Problem Set 1 that you had to read especially carefully?

Narrowing Down the Choices

Were there any of the questions from Problem Set 1 that you could have easily eliminated choices from?

Narrowing Down the Choices - Try this one

1. In the diagram, the area of rectangle $ABCD$ is 40. The area of $MBCN$ is



- (A) 15 (B) 10 (C) 30 (D) 12 (E) 16

Plug in the Given Values!

When $x = 9$, which of the following has the largest value?

- (A) \sqrt{x} (B) $\frac{x}{2}$ (C) $x - 5$ (D) $\frac{40}{x}$ (E) $\frac{x^2}{20}$

Plug in the Given Values!

If $a = 7$ and $b = 13$, the number of even positive integers less than ab is

(A) $\frac{ab - 1}{2}$

(B) $\frac{ab}{2}$

(C) $ab - 1$

(D) $\frac{a + b}{4}$

(E) $(a - 1)(b - 1)$

Try writing out a few cases!

When three consecutive positive integers are multiplied together, the answer is always

- (A) odd (B) a multiple of 6 (C) a multiple of 12
(D) a multiple of 4 (E) a multiple of 5

Try writing out a few cases!

The increasing list of five different integers $\{3,4,5,8,9\}$ has a sum of 33. How many increasing lists of five different single-digit positive integers have a sum of 33?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Work Time

Work on **Problem Set 2 and 3**.

For next week, check out the EWorkshop for whichever contest you are writing.