



Intermediate Math Circles

March 7, 2012

Problem Set: Linear Diophantine Equations II

1. Alyssa has a lot of mail to send. She wishes to spend exactly \$100 buying 49-cent and 53-cent stamps.
Parts (a)-(d), will step you through determining the number of ways she can do so.
Let's let x represent the number of 49-cent stamps she buys and y represent the number of 53-cent stamps she buys.
 - a) Use the Euclidean Algorithm to calculate $\gcd(49, 53)$.
 - b) Find a solution to $49x + 53y = 1$.
 - c) Find all solutions to $49x + 53y = 10000$.
 - d) Find all solutions to $49x + 53y = 10000$, where $x \geq 0$, $y \geq 0$.
 - e) List all of the ways that Alyssa can spend exactly \$100 buying 49-cent and 53-cent stamps.
2. At a museum, an adult ticket costs \$10 and a student ticket costs \$6. Suppose your school group spends exactly \$156 on tickets for a field trip to the museum. Determine all possibilities for the number of adults and students who went on the field trip.
3. Determine the number of ways you can make exactly \$200 using exactly 1000 coins if each coin is a quarter, a dime or a nickel.
4. Find the smallest positive integer x so that $157x$ leaves remainder 10 when divided by 24.
5. A person cashes a cheque at the bank. By mistake the teller pays the person the number of cents as dollars and the number of dollars as cents. The person spends \$3.50 before noticing the mistake, then after counting the money finds that there is exactly double the amount of the cheque. For what amount was the cheque drawn?