University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles
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Speedy, Speedy Arithmetics

Being able to perform arithmetics rapidly and accurately is an essential skill to succeed in higher mathematics. *(not to mention show off to your friends too!)* Previously, you have learned about divisibility rules and possible tricks to addition, subtraction as well as multiplication.

In today's circle, we'll focus, in addition to some new tricks, on commonly used strategies that help you perform speedy arithmetics in general.

But all in all, to become an arithmetics expert, the key is just to **PRACTICE, PRACTICE, PRACTICE!!!**

So let's get down to business!

Warm-up exercises:

Is 519647 divisible by 2?

Is 94632 divisible by 3?

Is 4673 divisible by 3?

Is 5166516651667253 divisible by 9?

A number is only divisible by 5 if its last digit is _____ or _____?

Calculate the following any way you like *(NO CALCULATORS!)*:

$56 + 24 + 19 + 21 + 48 + 922 + 86 =$

$37 \times 25 =$

$26 \times 19 =$

$82 \times 9 =$

$243 \times 11 =$

$18 \times 18 =$

$2566 \times 6 =$

$53^2 =$

$1475 \div 5 =$

$140 \times 5 =$

Multiply by 11

Multiplying a number by 10 is easy, you just add a '0' to the end of your original number.
Then multiplying a number by 11 is just multiplying that number by 10 and adding itself to it.

This is a quick way to do it:

1. Given a number, first put down its right most digit
2. Then add its right most digit with the digit on the left of it, put down the resulting *units* digit to the left of the digit you previously put down.
3. Record (remember) the carry over digit, if there is any.
4. Repeat step 2 for the second right most digit, adding in the previous carry over, if there is any, and so on, until you are at the left most digit (and so there are no more numbers to its left)
5. Add the carry over from previous addition of the second left most digit with the left most digit, if there is any, to this number, and put it down on the left most side of your result.
6. *Voila! You are done!*

Example:

$214352 \times 11 =$

$13856 \times 11 =$

Multiply by any number that ends with a relatively big number, say, 8 or 9

The strategy here is simple, simply multiply the number by a number that is relatively easy to get an answer, then subtract the original number a few times, depending on the difference between the number you multiplied by and the number you are originally supposed to multiply by.

Example:

$8624 \times 49 =$

$2133 \times 28 =$

Try this: Using the above, try to find a strategy that helps you when multiplying by a number that ends in a somewhat small number.

Multiplying or dividing a number by 5, 50, 500...etc.

5 is quite a special number, because it is exactly half of 10.

Multiplying by 5 is often easier when multiplied by 10, then divided by 2. Dividing by 5 is often easier when multiplied by 2, then divided by 10.

Example:

$142 \times 5 =$

$620 \div 5 =$

$6327 \times 5 =$

$39945 \div 5 =$

Try this: You can make your own rules for multiply/divide by 50, 500, etc.
Try this: Using the division strategy and try to divide a number that gives you a decimal when divided by 5 (ex. $4677 \div 5$*)*

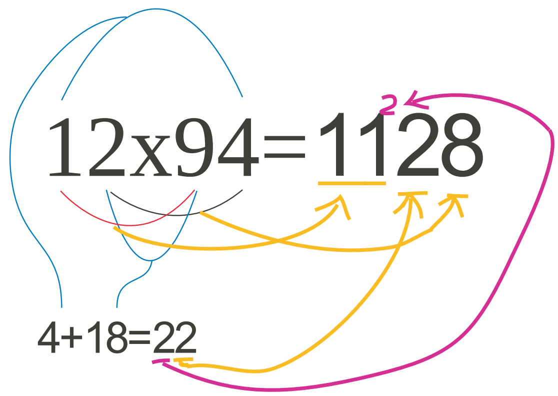
Multiplying two digit numbers

This is a very powerful method to learn.

Given any two 2 digit numbers, say $ab \times cd$, we can find its product by the following:

1. Multiply the right most digit of both numbers, put down the resulting product's *units* digit. Remember any carry overs.
2. Now multiply the right most digit of the **first** number with the left most digit of the **second** number, remember this product.
3. Now multiply the right most digit of the **second** number with the left most digit of the **first** number, add this product to the product you obtained from step 2) as well as add in any carry overs from step 1), put down the *units* digit of this resulting number on the left of the first number you put down. Remember any carry overs.
4. Multiply the left most digit of both numbers, add this product with any carry overs from step 4). Put down this entire number on the left of the number you previously put down.
5. *Voila! You are done!*

Examples:



$12 \times 14 =$

$34 \times 83 =$

$25 \times 16 =$

$98 \times 42 =$

<u>**Addition and Subtraction**</u>

These will become natural as you practice more and more arithmetics.

Addition and Subtraction are often faster when done in ***2 or more steps*** instead of 1:

Adding a number to another number is much faster if you add the closest number that ends in 0, then add/subtract the difference between the original number you are supposed to add and the number that you added.

The same goes for subtraction, except that you would subtract, then add/subtract back the difference.

When you are adding a lot of numbers together, for every digit, group that digit's numbers in a way such that they add up to 10 and carry the number of groups of 10 over, write down sum of that digit of any remaining ungrouped numbers, remember any additional carry overs.

Examples:

$427 + 39 =$

$517 - 38 =$

$56 + 197 =$

$926470 - 602 =$

$946 - 168 =$

$1629 + 94571 =$

Squaring a number

Background knowledge:

We write a number a multiplied by itself this way: $a \times a$

Another way to write a number a multiplied by itself is: $a \times a = a^2$

This is called **"a to the *exponent* of 2"** where the 2 denotes the number of times we multiply the number.

Example:

61×61 can be written as 61^2

So then how should I write $3 \times 3 \times 3 \times 3 \times 3$?

Today, we are only interested in something to the exponent of 2. (i.e. a number multiplied by itself)

Let's get started!

As with multiplication of single digit numbers, the squares of numbers from 1 to 20 should be memorized! (Exercises 67-76)

Remember, mathematics does not only depend on logic and reasoning; it is also reliant on knowing your facts, memorizing your methods and the ability to apply them!

Speed arithmetics requires some memorization of basic arithmetics too!

Squaring any number that ends with 5

Now moving on to working with some squares, for finding the square of any 2 digit number, the multiplication rule is the fastest aside from memorizing.

But for more than 2 digits, we have some neat tricks.

A number A that ends with 5 can be written as follows: $A = 10 \times a + 5$ where a is A with the right most digit (i.e. the 5 on the right) dropped.

Then we can find the square of A, i.e. A^2 by:

- Writing down the product of $a \times (a + 1)$
- Appending 25 to the right of the product that you got.

Example:

$135^2 =$

$1015^2 =$

Squaring any number

While with the popularization of calculators this method might seem out of date, it's still a catchy little trick to keep in your pocket.

You can find the square of any number by:

1. Take a number (close to the number that you want to square) whose square can be computed easily, this is your first number
2. Take the difference of the number minus your original number, call this difference d
3. Multiply the difference by -1, add it to your original number to get a second number
4. Multiply the first number and the second number to get a product
5. Add the *square of* d to your product
6. *Voila! You are done!*

Example:

$2012^2 =$

$131^2 =$

Exercises:

1). Is 5125 divisible by 2?

2). Is 2262 divisible by 2?

3). Is 5214 divisible by 3?

4). Is 99999993331111113 divisible by 9?

5). For what value of a is $132a4$ divisible by 3?

6). List all numbers between 234 and 266 that are divisible by 5.

7). $3142 \times 11 =$

8). $516 \times 11 =$

9). $675 \times 11 =$

10). $3198 \times 11 =$

11). $11 \times 3338 =$

12). $649957 \times 11 =$

13). $11 \times 40022 =$

14). $110 \times 77425 =$

15). $44264 \times 22 =$

16). $971263 \times 121 =$

17). $10132 \times 99 =$

18). $443 \times 51 =$

19). $1964 \times 102 =$

20). $32234 \times 211 =$

21). $548 \times 38 =$

22). $42787 \times 690 =$

23). $12 \times 13 =$

24). $15 \times 21 =$

25). $23 \times 32 =$

26). $32 \times 68 =$

27). $81 \times 12 =$

28). $83 \times 74 =$

29). $53 \times 24 =$

30). $76 \times 46 =$

31). $55 \times 71 =$

32). $35 \times 34 =$

33). $82 \times 89 =$

34). $45 \times 67 =$

35). $61 \times 14 =$

36). $41 \times 77 =$

37). $27 \times 29 =$

38). $78 \times 53 =$

39). $63 \times 54 =$

40). $27 \times 132 =$

41). $77 \times 4200 =$

42). $3400 \times 520 =$

43). $49 \times 682 =$

44). $792 \times 473 =$

45). $306 \times 5 =$

46). $122 \times 5 =$

47). $1898 \times 50 =$

48). $500 \times 6337 =$

49). $477 \times 25 =$

50). $81664 \times 125 =$

51). $830 \div 5 =$

52). $955 \div 5 =$

53). $1100 \div 25 =$

54). $4425 \div 5 =$

55). $68 + 98 =$

56). $109 + 2447 =$

57). $6043 + 74 =$

58). $789 + 652 =$

59). $999 + 888 =$

60). $6116 + 10547 =$

61). $300 - 102 =$

62). $72845 - 693 =$

63). $226 - 87 =$

*64). $1102 - 6106 =$

65). $49772 + 693 + 807 =$

66). $1288 - 503 + 623 =$

67). $11^2 =$

68). $14^2 =$

69). $13^2 =$

70). $15^2 =$

71). $18^2 =$

72). $19^2 =$

73). $17^2 =$

74). $12^2 =$

75). $20^2 =$

76). $16^2 =$

77). $45^2 =$

78). $75^2 =$

79). $105^2 =$

80). $325^2 =$

81). $403^2 =$

82). $79989^2 =$

83). $187^2 =$

84). $2011^2 =$

*85). $\frac{5}{6} + \frac{1}{2} + \frac{10}{12} + \frac{1}{3} + \frac{5}{6} + \frac{1}{6} + \frac{1}{3} =$

*86). $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{5} \times \frac{5}{8} \times \frac{8}{13} \times \frac{13}{21} \times \frac{21}{22} =$

*87). Marie purchased six bags of apples from the store at a price of $3.69 a bag. When she arrived home, she realized that they were on sale for $3.19 a bag, so she returned to the store to get a refund of the difference. The cashier gave her the difference in quarters. — How many quarters did Marie receive?

*88). (Exponents, dimension conversion) *Moore's law* predicts that a computer will double in processing speed every 18 months. In 2012, it took my computer *35 minutes* to digitally render a very fancy hand-drawn picture. Approximately how many *hours* would this rendering have taken back in 2000?