Grade 7/8 Math Circles
February 15th, 2012
Geometric Constructions Exercise Answers

2). B).
The triangle you've constructed is equilateral because PR and RQ are radius of the two identical circles and hence they are equal. Moreover, PQ is also equal to the radius of these circles. Hence I have a triangle whose 3 sides are equal — an equilateral triangle.

4). B).
The construction of a triangle requires that the length of any 2 sides combined must be longer than the 3rd side.
The figure is impossible to construct because the side PQ is longer than the combined length of RQ and PR.

6). C). Consult the following diagram below. The line OP is a radius of the circle, so is the line OA. Therefore, the length of AB is equal to the length of PQ (AOB and POQ are two isosceles triangles with the same side lengths and have $\angle O = 90°$, hence they are congruent triangles, so their respective bases must also equal, i.e. $AB = PQ$). I may rotate the inside square $ABCD$ 45° (either direction) to have my new vertices of the square sit on the points $PQRS$. The lines PR and QS divides the large square $EFGH$ exactly into four pieces while my rotated square $ABCD$ takes up exactly the inside triangle of each piece — half. Hence square $EFGH$ has twice the area as square $ABCD$.