University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles
March 21, 2012
Circuits

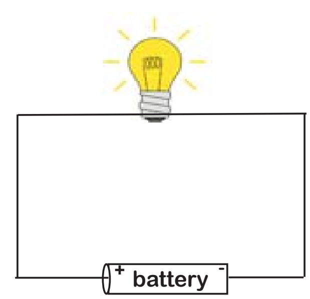
What is electricity?

Electricity is a type of energy that comes in two forms. When electricity gathers in one place, it is known as **static electricity**. One example of static electricity is when you rub a balloon against your shirt and it sticks to your body.
The other type of electricity is called **current electricity** which is when it moves from one place to another like when electric current flows through a wire from a battery to a light bulb.

Today we are going to look at current electricity and some of the mathematics involved in electric circuits.

Electric Circuits

A **circuit** is a closed path around which electric current flows. An example of a circuit would be wires connecting a battery to a light bulb.



Current is a steady flow of electrons that moves throughout the circuit. It is measured in amperes (or amps) and represented by the symbol "I".

Voltage is the electrical force that makes the electric current move through the wire. We represent voltage using a "V" and it is measured in volts.

Resistance is a property by which the passage of current is opposed. Resistance causes some of the electric energy to be transformed into heat. Resistance is represented by "R" and measured in ohms (Ω).

Ohm's Law:

Ohm's law describes the relationship between voltage, current, and resistance.

$$\begin{aligned} \text{Voltage} &= \text{Current} \times \text{Resistance} \\ V &= I \times R \end{aligned}$$

We can use Ohm's law to determine the current, voltage or resistance of any battery or light bulb in our circuits as long as we know two of the other quantities.

Circuit problem solving strategies:

- Write down what quantities are known and what is unknown
- Write down what you are trying to find
- Write out what equations will be used
- Draw a diagram

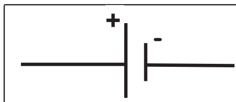
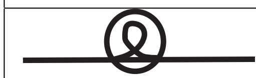
Example:

1) In a simple circuit, 10 amps of electric current pass through a light bulb whose resistance is 2 Ω. Determine the voltage at the light bulb.
I = 10 amps
R = 2 Ω
V = ?

$$\begin{aligned} V &= I \times R \\ V &= 10 \times 2 \\ V &= 20 \text{ volts} \end{aligned}$$

Circuits:

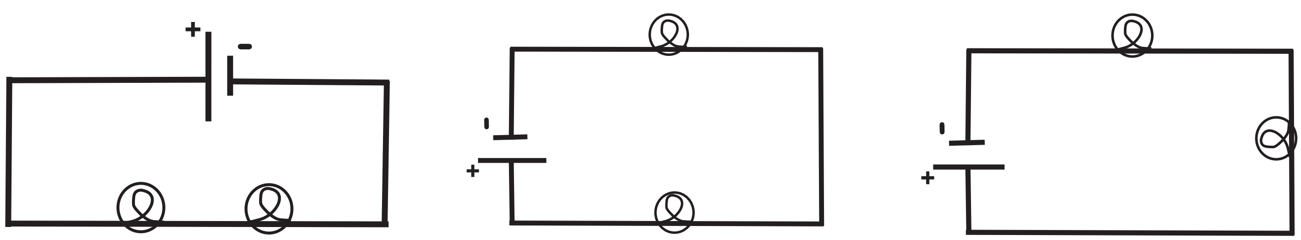
Common Symbols:

	Battery
	Light Bulb
	Switch

Light Bulbs Connected in Series:

One way that we can connect light bulbs in a circuit is in series, which means that the current flows through the light bulbs one after another.

In the diagrams below, the light bulbs are connected in series.



For light bulbs in series, the following three properties are true.

1. The total current in the circuit (ie. the total current leaving the battery) is equal to the current flowing through each of the light bulbs in the circuit. (ie. $I_{total} = I_1 = I_2 = ...$)

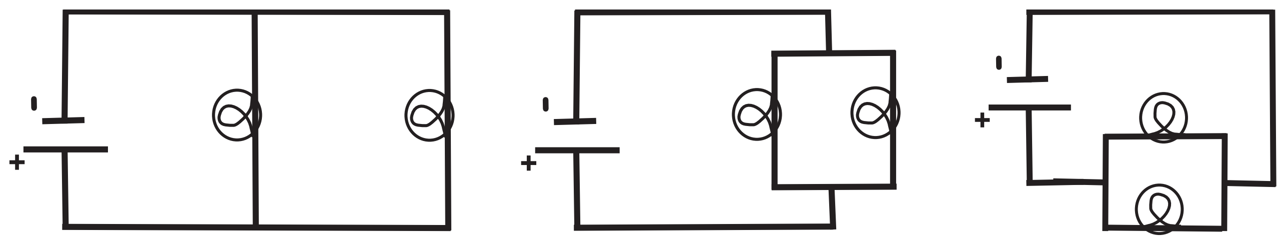
2. The total voltage in the circuit (ie. the total voltage leaving the battery) is equal to the sum of voltage at each of the light bulbs. (ie. $V_{total} = V_1 + V_2 + ...$)

3. The total resistance in the circuit (ie. the total resistance at the battery) is equal to the sum of resistance at each of the light bulbs. (ie. $R_{total} = R_1 + R_2 + ...$)

Light Bulbs Connected in Parallel:

Another way that we can connect light bulbs in a circuit is in parallel, which means that the current branches out at a single point (called a node) and join up at another point somewhere else in the circuit.

In the diagrams below, the light bulbs are connected in parallel.



For light bulbs that are parallel in a circuit, the following three properties are true.

1. The total current in the circuit (ie. the total current leaving the battery) is equal to the sum of the current flowing through each of the light bulbs in the circuit.
(ie. $I_{total} = I_1 + I_2 + ...$)

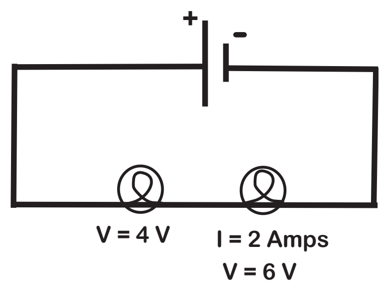
2. The total voltage in the circuit (ie. the total voltage leaving the battery) is equal to the voltage at each of the light bulbs in the circuit.
(ie. $V_{total} = V_1 = V_2 = ...$)

3. The inverse of the total resistance in the circuit (ie. the total resistance at the battery) is equal to the sum of inverses of the resistance at each of the light bulbs.
(ie. $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$)

Examples:

1) For each of the diagrams below, determine if the light bulbs are connected in series with each other, or in parallel with each other. Then, determine the total current, voltage, and resistance leaving the battery in each circuit.

a)



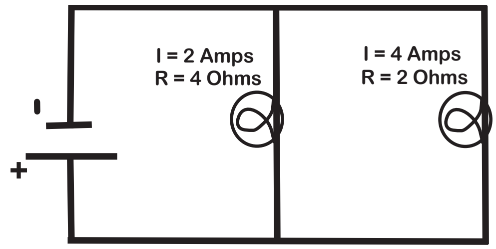
In this circuit, the light bulbs are connected in series.
Since they are connected in series, we know that the current is the same throughout the circuit. This means that since we know that the current at one of the light bulbs is 2 Amps, we know that the total current is also 2 Amps.
Then, we can determine the total voltage by adding the voltages from each of the batteries. So,

$$\begin{aligned} V_{Total} &= 4V + 6V \\ V_{Total} &= 10V \end{aligned}$$

Finally, we can determine the total resistance at the battery by using Ohm's law and the two other quantities that we have already found.

$$\begin{aligned} V_{Total} &= I_{Total} \times R_{Total} \\ R_{Total} &= \frac{V_{Total}}{I_{total}} \\ R_{Total} &= \frac{10V}{2A} \\ R_{Total} &= 5\Omega \end{aligned}$$

b)



In this circuit, the light bulbs are connected in parallel.
We can determine the total voltage by determining the voltage at any of the light bulbs since it is the same at any point in the circuit. Let's determine the voltage at the first light bulb using Ohm's Law:

$$\begin{aligned} V_{Total} = V_1 &= I_1 \times R_1 \\ V_{Total} &= 2A \times 4\Omega \\ V_{Total} &= 8V \end{aligned}$$

Then we can determine the total current in the circuit since it is the sum of the current at each light bulb. So,

$$\begin{aligned} I_{Total} &= 2A + 4A \\ I_{Total} &= 6A \end{aligned}$$

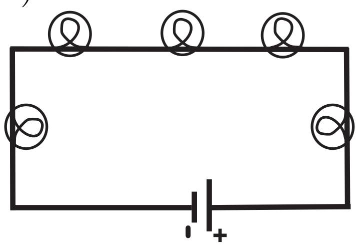
Finally, we can determine the total resistance since we know the resistance at each of the batteries.

$$\begin{aligned} \frac{1}{R_{Total}} &= \frac{1}{R_1} + \frac{1}{R_1} \\ \frac{1}{R_{Total}} &= \frac{1}{4} + \frac{1}{2} \\ \frac{1}{R_{Total}} &= \frac{3}{4} \\ 3 \times R_{Total} &= 4 \\ R_{Total} &= \frac{4}{3}\Omega \end{aligned}$$

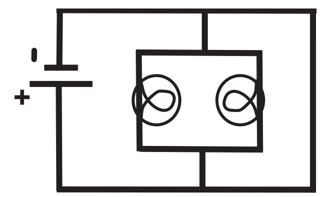
Exercises:

1. For each of the diagrams below, determine if the light bulbs are in series with each other, or in parallel with each other.

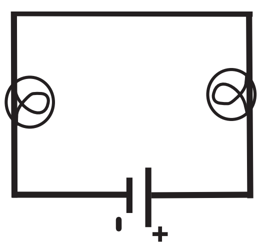
a)



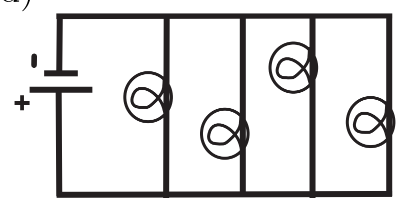
b)



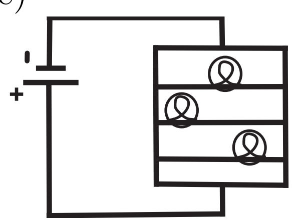
c)



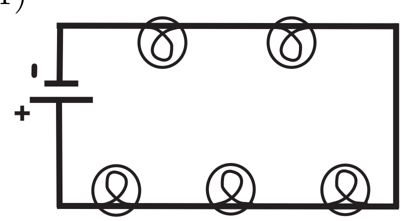
d)



e)

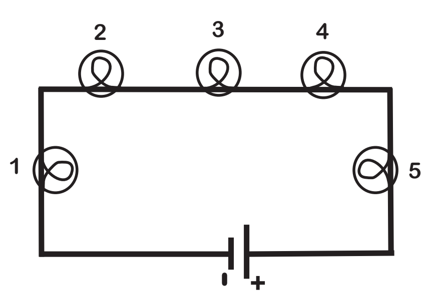


f)



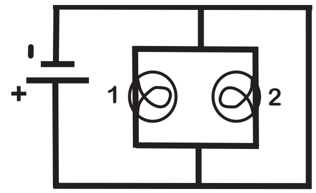
2. For each of the diagrams and corresponding tables below, fill in the missing values in the table using Ohm's Law and circuit properties.

a)



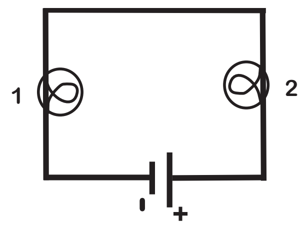
$V_{total} =$	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$I_{total} = 5\text{A}$	$I_1 =$	$I_2 =$	$I_3 =$	$I_4 =$	$I_5 =$
$R_{total} =$	$R_1 = 1\Omega$	$R_2 = 3\Omega$	$R_3 = 2\Omega$	$R_4 = \frac{3}{2}\Omega$	$R_5 = \frac{1}{2}\Omega$

b)



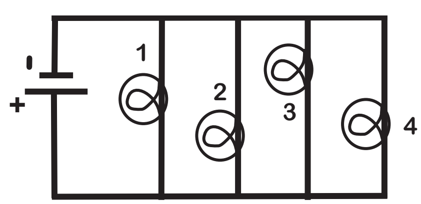
$V_{total} =$	$V_1 =$	$V_2 = 4\text{V}$
$I_{total} =$	$I_1 = 2\text{A}$	$I_2 =$
$R_{total} =$	$R_1 = 2\Omega$	$R_2 = 2\Omega$

c)



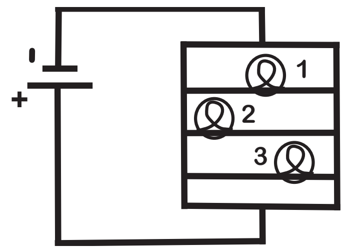
$V_{total} = 18\text{V}$	$V_1 =$	$V_2 =$
$I_{total} =$	$I_1 = 3\text{A}$	$I_2 =$
$R_{total} =$	$R_1 =$	$R_2 = 4\Omega$

d)



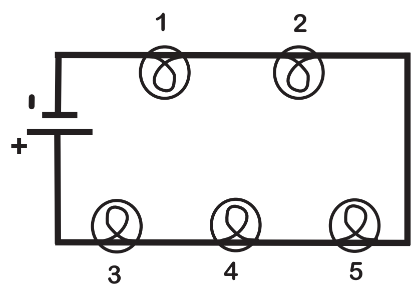
$V_{total} =$	$V_1 =$	$V_2 =$	$V_3 = 12\text{V}$	$V_4 =$
$I_{total} =$	$I_1 =$	$I_2 =$	$I_3 =$	$I_4 =$
$R_{total} =$	$R_1 = 4\Omega$	$R_2 = 2\Omega$	$R_3 = 3\Omega$	$R_4 = 1\Omega$

e)



$V_{total} =$	$V_1 =$	$V_2 =$	$V_3 =$
$I_{total} =$	$I_1 =$	$I_2 = 2\text{A}$	$I_3 =$
$R_{total} = 2\Omega$	$R_1 = 4\Omega$	$R_2 = 8\Omega$	$R_3 =$

f)



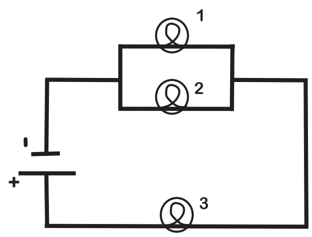
$V_{total} =$	$V_1 =$	$V_2 = 16\text{V}$	$V_3 =$	$V_4 =$	$V_5 = 28\text{V}$
$I_{total} =$	$I_1 =$	$I_2 = 4\text{A}$	$I_3 =$	$I_4 =$	$I_5 =$
$R_{total} = 25\Omega$	$R_1 = 3\Omega$	$R_2 =$	$R_3 = 5\Omega$	$R_4 =$	$R_5 =$

Challenge Problems:

1. For each of the diagrams and corresponding tables below, fill in the missing values in the table using Ohm's Law and circuit properties.

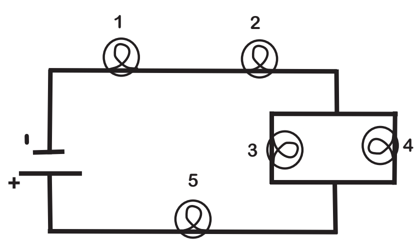
   Note: All of the circuits below contain both parallel and series components. To solve for the missing values, you will need to solve for the total values in the parallel component. Then you can simplify the circuit by treating the parallel component as a single light bulb in series with the rest of the circuit. The values for current, voltage, and resistance of the simplified light bulb will be the total values of the parallel component.

   a)



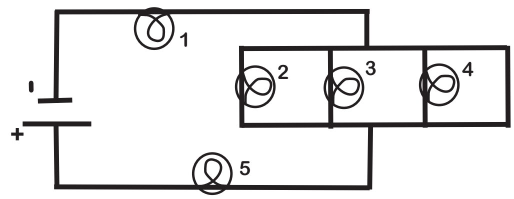
$V_{total} =$	$V_1 = 6\text{V}$	$V_2 =$	$V_3 =$
$I_{total} =$	$I_1 =$	$I_2 =$	$I_3 = 2\text{A}$
$R_{total} =$	$R_1 = 12\Omega$	$R_2 = 4\Omega$	$R_3 = 4\Omega$

   b)



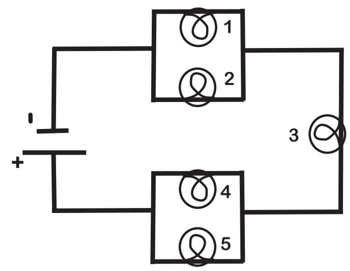
$V_{total} =$	$V_1 =$	$V_2 = 3\text{V}$	$V_3 =$	$V_4 = 6\text{V}$	$V_5 =$
$I_{total} = \frac{3}{2}\text{A}$	$I_1 =$	$I_2 =$	$I_3 = 1\text{A}$	$I_4 = \frac{1}{2}\text{A}$	$I_5 =$
$R_{total} =$	$R_1 = 4\Omega$	$R_2 =$	$R_3 =$	$R_4 =$	$R_5 = 2\Omega$

c)



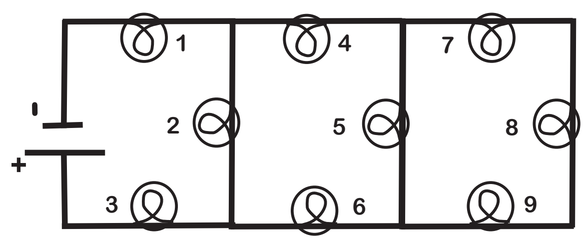
$V_{total} =$	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$I_{total} = 5\text{A}$	$I_1 =$	$I_2 =$	$I_3 =$	$I_4 =$	$I_5 =$
$R_{total} =$	$R_1 = 5\Omega$	$R_2 = 2\Omega$	$R_3 = 3\Omega$	$R_4 = 4\Omega$	$R_5 = \frac{1}{13}\Omega$

d)



$V_{total} = 25\text{V}$	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$I_{total} =$	$I_1 =$	$I_2 =$	$I_3 =$	$I_4 =$	$I_5 =$
$R_{total} =$	$R_1 = 10\Omega$	$R_2 = 4\Omega$	$R_3 = 3\Omega$	$R_4 = 8\Omega$	$R_5 = 1\Omega$

2. a) Each light bulb in the diagram has resistance R. Determine the total resistance of the circuit in terms of R.



b) Given that $R_T = 15\Omega$ and $I_T = 2A$, use the total resistance found in part (a) to fill in all the missing values in the table below.

$V_{total} =$	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$I_{total} =$	$I_1 =$	$I_2 =$	$I_3 =$	$I_4 =$	$I_5 =$
$R_{total} =$	$R_1 =$	$R_2 =$	$R_3 =$	$R_4 =$	$R_5 =$
	$V_6 =$	$V_7 =$	$V_8 =$	$V_9 =$	
	$I_6 =$	$I_7 =$	$I_8 =$	$I_9 =$	
	$R_6 =$	$R_7 =$	$R_8 =$	$R_9 =$	