



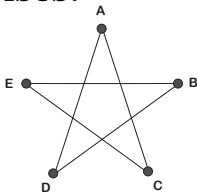
Grade 7/8 Math Circles

March 7, 2012

Introduction to Graph Theory

Exercises:

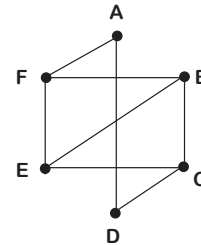
1)



The points of a star are labeled A,B,C,D,E.

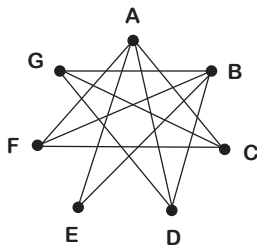
- a) $\{(A,C), (C,E), (E,B), (B,D), (D,A)\}$
 b) $\{A,D,B\}$
 c) Yes, the graph is connected.
 d) Cycle: $\{A,C,E,B,D,A\}$ - Yes, this cycle contains all of the vertices.

- 2) a) $\{(A,D), (D,C), (C,E), (E,B), (E,F), (F,A), (F,B), (B,C)\}$
 b) Shortest path: $\{F,A,D\}$, longest path: $\{F,B,E,C,D\}$
 c) Yes, the graph is connected.
 d) One possible cycle: $\{C, E, F, B, C\}$,
 Cycle containing all of the vertices: $\{C, D, A, F, B, E, C\}$



The graph above has vertices labeled A,B,C,D,E,F.

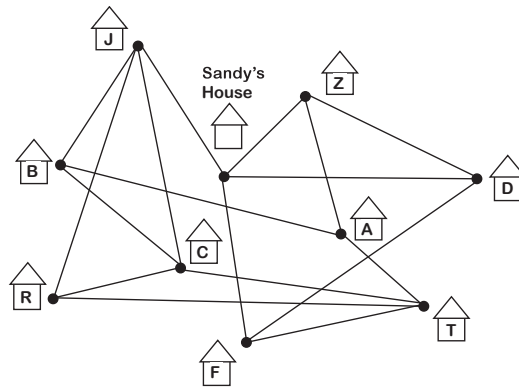
3)



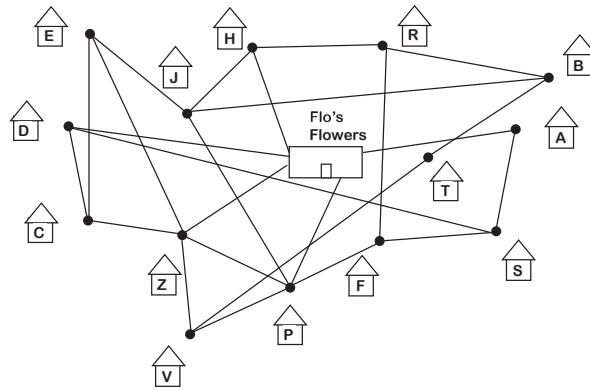
The above graph has vertices labeled A,B,C,D,E,F,G.

- a) This graph has 11 edges. They are: $\{(A,C), (A,D), (A,E), (A,F), (B,G), (B,F), (B,E), (B,D), (G,C), (G,D), (F,C)\}$
 b) Shortest path: $\{A,D,G\}$, longest path: $\{A,E,B,F,C,G\}$
 c) Yes, the graph is connected.
 d) One possible cycle: $\{B,F,C,G,B\}$,
 Cycle containing all of the vertices: $\{B,F,C,G,D,A,E,B\}$

4) Sandy should visit the houses in the following order: {Sandy's, D,F,T,R,C,J,B,A,Z, Sandy's}



5) We would like to find a cycle that starts (and ends) at Flo's Flowers. One possibility is: {Flo's, H,R,F,P,Z,V,T,B,J,E,C,D,S,A,Flo's}

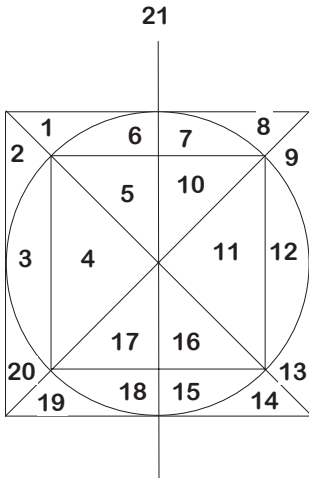


Maps and Colouring:

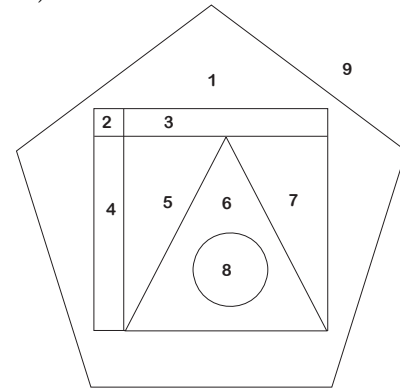
Exercises:

1) Determine how many regions each of the following maps have.

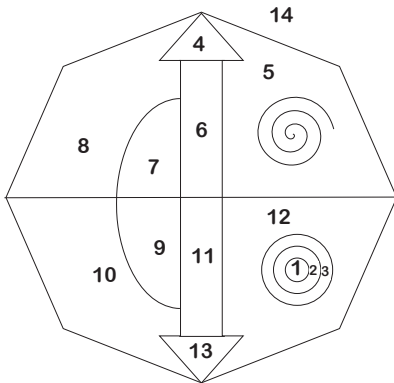
a)



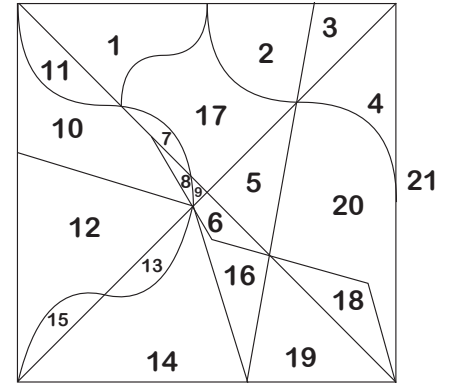
b)



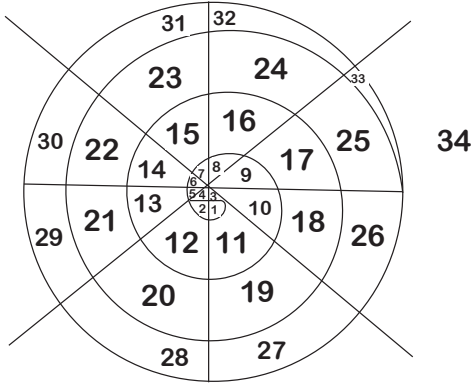
c)



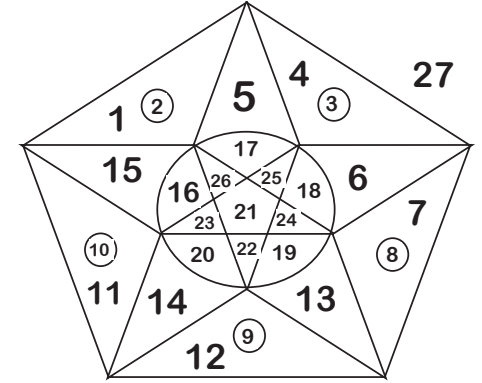
d)



e)

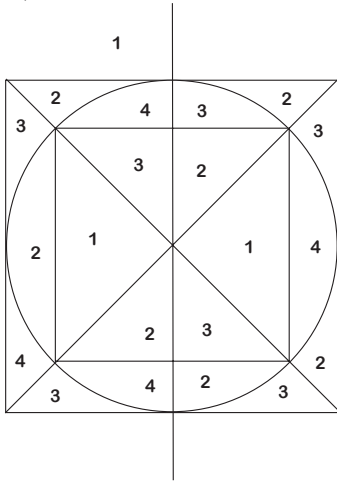


f)

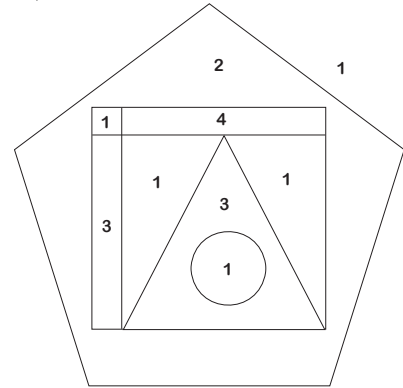


2) Colour each of the maps in exercise 1 using at most 4 colours.

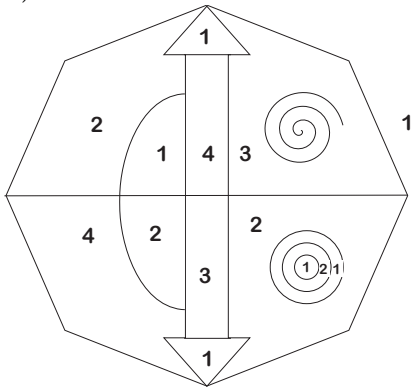
a)



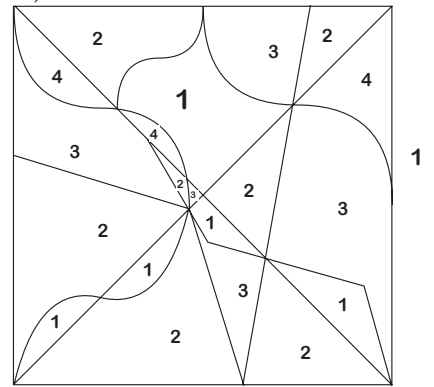
b)



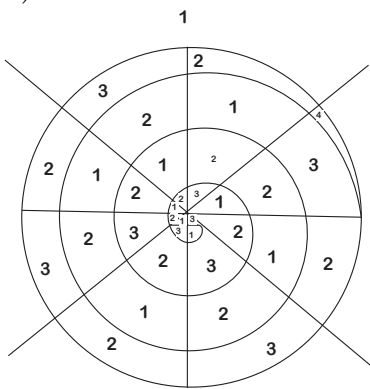
c)



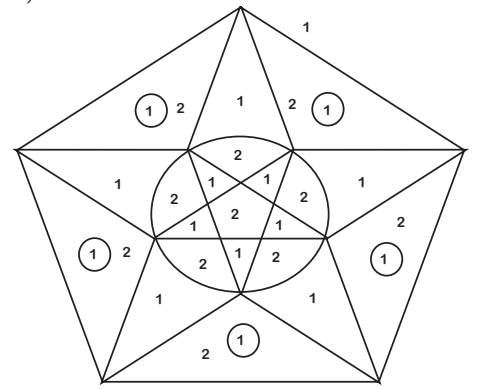
d)



e)



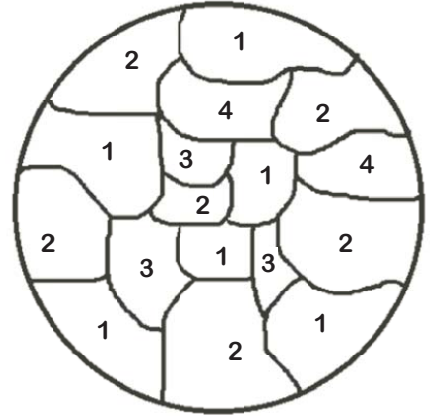
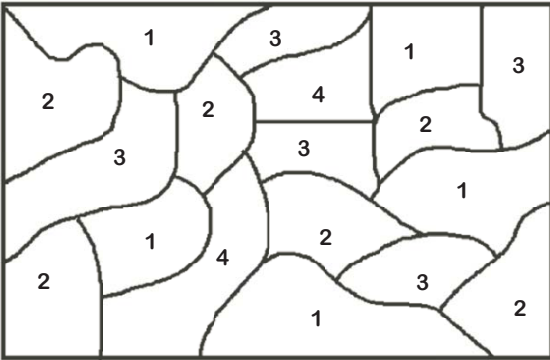
f)



3) Colour each of the following using at most 4 colours.

b)

a)



c)

