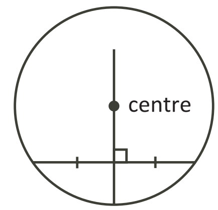
Intermediate Math Circles
Wednesday November 07 2012
Problem Solving - Preparing for Contests

Recap from last Week

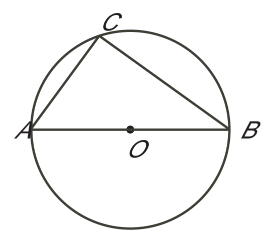
The Chord Right Bisector Property

- The right bisector of a chord passes through the centre of the circle.
- The perpendicular from the centre to a chord bisects the chord.
- The line joining the centre to the midpoint of a chord is perpendicular to the chord.
- The centre of a circle is the intersection of the right bisectors of two non-parallel chords.



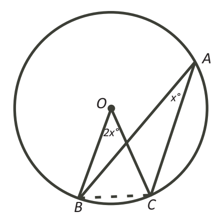
Angle in a Semicircle Property

- An angle inscribed in a semicircle is $90°$. In this case, if AB is a diameter, then $\angle ACB = 90°$.



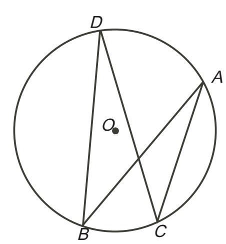
Angle at the circumference Property

An angle at the centre of a circle is twice the angle at the circumference standing on the same (side of a common) chord.



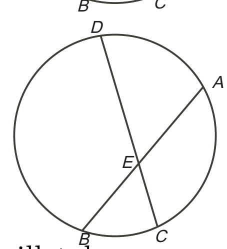
Angles Inscribed in a Circle by a Common Chord

Two angles inscribed in a circle and standing on the same side of a common chord are equal.



Chord Splitting Property

If two chords in a circle intersect, the product of the lengths of the two parts of one chord is equal to the product of the lengths of the two parts of the other chord. In this case, $DE \times EC = AE \times EB$.



Go to the Euclid Contest eWorkshop. There is a circle geometry package that will take you further with circles. Go to

http://cemc.math.uwaterloo.ca/contests/euclid_eWorkshop.html

and work through the material.

Writing Multiple Choice Contests

- Read the problem carefully.
- Can I rule out any answers?
- Can I use the answers to help solve the problem?
- Is it reasonable to guess?

Examples

If $\frac{2x}{2.5} = 0.004$, then the value of x is

(A) 0.05 **(B)** 0.005 **(C)** 0.0008 **(D)** 0.002 **(E)** 0.5

Since $\frac{2}{2.5} < 1$, then $\frac{2}{2.5}x < x$ and we can rule out both (C) and (D) since they are too small. It is possible to easily reason that 0.5 is too big. Very quickly, we have ruled out 3 of the 5 answers. Often removing choices helps us to zero in on the correct answer and possibly how to get it. The correct answer, by the way, is B.

The value of $\sqrt{36+64} - \sqrt{25-16}$ is

(A) 5 **(B)** 7 **(C)** 13 **(D)** 11 **(E)** 9

Since $36 + 64 = 100$ and $\sqrt{100} = 10$ the answer must be less than 10. Answers C and D can be removed. The correct answer is B.

The value of $(\sqrt{100} - \sqrt{36})^2$ is

(A) 16 **(B)** 256 **(C)** 8 **(D)** 1024 **(E)** 4096

Since $\sqrt{100} = 10$ and $10^2 = 100$, the answer must be less than 100. Answers B, D and E can be removed. The correct answer is A. By eliminating answers, the probability of getting the question correct is 50%.

If $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{n}{12} = 2$, the value of n is

(A) -4 **(B)** 13 **(C)** 18 **(D)** 4 **(E)** 1

In this problem, the solver could simply substitute each answer until obtaining the correct answer from the list. The correct answer is E.

If $a = 7$ and $b = 13$, the number of even positive integers less than ab is

(A) $\dfrac{ab-1}{2}$ **(B)** $\dfrac{ab}{2}$ **(C)** $ab-1$ **(D)** $\dfrac{a+b}{4}$ **(E)** $(a-1)(b-1)$

$a \times b = 91$ and so we want to know how many even positive integers are less than 91. The numbers are $\{2, 4, 6, \cdots, 88, 90\}$, 45 numbers in total. So now we could test all of the expressions to determine the correct result. The correct answer is A. If you think about it, 90 is 1 less than 91 and half of the numbers are even. $\frac{ab-1}{2}$ makes sense.

At Math Circles we worked on multiple choice problems and full solution problems during our session.

For students getting this package from the website, I am attaching a sample contest which requires full solutions and I am attaching complete solutions to the sample contest.

For practise writing multiple choice contests go to

http://cemc.math.uwaterloo.ca/contests/past_contests.html

You will find a good variety of multiple choice contests and solutions.

The Gauss contest is written by grade 7 and 8 students, the Pascal contest is written by grade 9 students, the Cayley contest is written by grade 10 students and the Fermat contest is written by grade 11 students. These contests may also be written by students in grades below the target grade of the contest.

You will also find past full solution contests (and solutions).

The Canadian Intermediate Math Contest is written by grade 9 and 10 students, the Canadian Senior Math Contest is written by grade 11 and 12 students, the Fryer Contest is written by grade 9 students, the Galois Contest is written by grade 10 students, the Hypatia Contest is written by grade 11 students and the Euclid Contest is written by grade 12 students. These contests may also be written by students in grades below the target grade(s) of the contest.

There is a wealth of resources for students to enjoy. Happy problem solving.