



Intermediate Math Circles
Wednesday November 21 2012
Problem Set 7

1. The point $(a, 2)$ is the point of intersection of the lines with equations $y = 2x - 4$ and $y = x + k$. Determine the value of k .
2. Graph the following regions.
 - a) $y \leq -2$
 - b) $x > 3$
 - c) $y \geq 2x - 5$
 - d) $2x + y < 4$
3. To find x -intercepts, set $y = 0$ and solve for x . To find y -intercepts, set $x = 0$ and solve for y . Graph the following regions by finding intercepts.
 - a) $3x - 4y > 12$
 - b) $5x + 3y \leq 5$
4. Graph the feasible region given the following inequalities:

$$\begin{aligned}x + y &\leq 9 \\x + 2y &\leq 15 \\2x + y &\leq 15 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

5. Graph the feasible region given the following inequalities:

$$\begin{aligned}x + 2y &\geq 6 \\2x + y &\geq 5 \\2x + 3y &\geq 10 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

6. The correct formula for converting Celsius temperature C to a Fahrenheit temperature F is given by $F = \frac{9}{5}C + 32$.

Andrew does not like arithmetic. So he approximates the Fahrenheit temperature by doubling C and then by adding 30 to get f .

If $f < F$, then the error in the approximation is $F - f$; otherwise, the error in the approximation is $f - F$. Determine the largest possible error in the approximation that Andrew would make when converting Celsius temperatures C with $20 \leq C \leq 35$.

7. Gloria is trying to devise a strategy to earn the highest return on her investments. She estimates that investing in real estate yields a 13% annual return on the investment, and the stock market a 17% return. Eeshan does some calculations and advises Gloria to invest at least as much in real estate as in stocks. If she has \$20,000 to invest, how should she invest it? Set up the inequalities that satisfy the given conditions.
8. Suppose that x and y are positive numbers with

$$\begin{aligned}xy &= \frac{1}{9} \\x(y+1) &= \frac{7}{9} \\y(x+1) &= \frac{5}{18}\end{aligned}$$

What is the value of $(x+1)(y+1)$?

9. The line $y = -\frac{3}{4}x + 9$ crosses the x -axis at P and the y -axis at Q . Point $T(r, s)$ is on line segment \overline{PQ} . If the area of $\triangle POQ$ is three times the area of $\triangle TOP$, then what is the value of $r + s$?
10. A triangle has vertices $A(0, 3)$, $B(4, 0)$, $C(k, 5)$, where $0 < k < 4$. If the area of the triangle $\triangle ABC$ is 8, determine the value of k .