

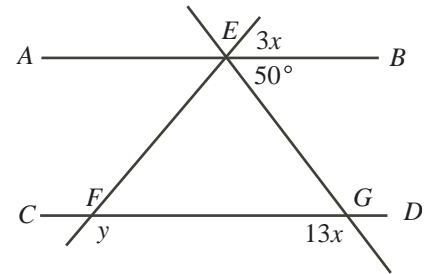


# Intermediate Math Circles

## Wednesday 10 October 2012

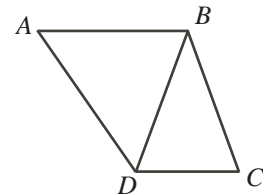
### Problem Set 1

1. In the diagram,  $AB$  is parallel to  $CD$ . Determine the values of  $x$  and  $y$ .

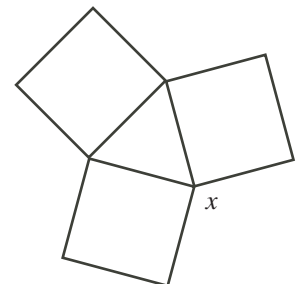


2. Triangle  $ABC$  has a right angle at  $B$ .  $AC$  is extended to  $D$  so that  $CD = CB$ . The bisector of angle  $A$  meets  $BD$  at  $E$ . Prove that  $\angle AEB = 45^\circ$ .

3. In the diagram,  $AB$  is parallel to  $DC$  and  $AB = BD = BC$ . If  $\angle A = 52^\circ$ , determine the measure of  $\angle DBC$ .

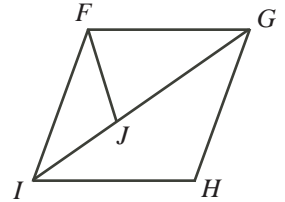


4. The diagram shows three squares of the same size. What is the value of  $x$ ?



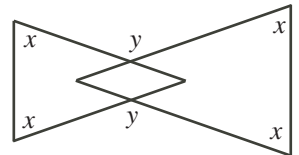


5. The diagram shows a rhombus  $FGHI$  and an isosceles triangle  $FGJ$  in which  $GF = GJ$ . Angle  $FJI$  equals  $111^\circ$ . What is the measure of angle  $JFI$ ?

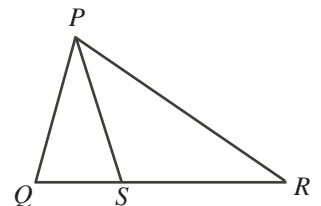


6.  $ABCD$  is a square. The point  $E$  is outside the square so that  $CDE$  is an equilateral triangle. Find angle  $BED$ .

7. The diagram shows two isosceles triangles in which the four angles marked  $x$  are equal. The two angles marked  $y$  are also equal. Find an equation relating  $x$  and  $y$ .

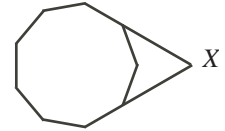


8. In the diagram,  $QSR$  is a straight line.  $\angle QPS = 12^\circ$  and  $PQ = PS = RS$ . What is the size of  $\angle QPR$ ?

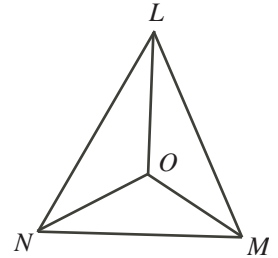




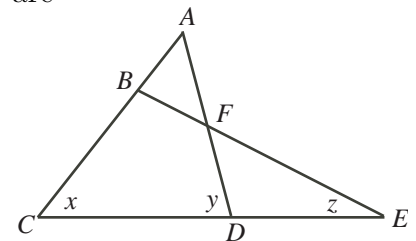
9. The diagram shows a regular nonagon with two sides extended to meet at point  $X$ . What is the size of the acute angle at  $X$ ?



10. The three angle bisectors of triangle  $LMN$  meet at a point  $O$  as shown. Angle  $LMN$  is  $68^\circ$ . What is the size of angle  $LOM$ ?



11. In the figure shown,  $AB = AF$  and  $ABC$ ,  $AFD$ ,  $BFE$ , and  $CDE$  are all straight lines. Determine an equation relating  $x$ ,  $y$  and  $z$ .

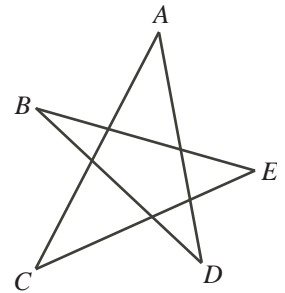


12. The angles of a nonagon are nine consecutive numbers. What are these numbers?



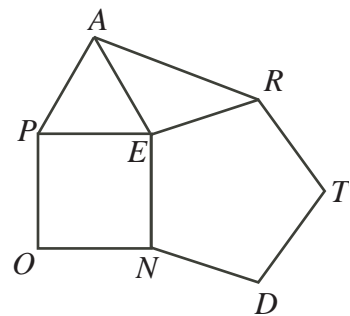
13. What is the measure of the angle formed by the hands of a clock at 9:10?

14. Determine the sum of the angles  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$  in the five-pointed star shown.



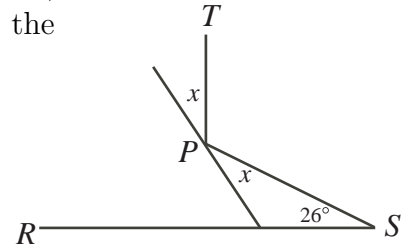
15. In  $\triangle PQR$ ,  $PQ = PR$ .  $PQ$  is extended to  $S$  so that  $QS = QR$ .  
Prove that  $\angle PRS = 3(\angle QSR)$ .

16. A regular pentagon is a five-sided figure which has all of its angles equal and all of its side lengths equal. In the diagram,  $TREND$  is a regular pentagon,  $PEA$  is an equilateral triangle, and  $OPEN$  is a square. Determine the size of  $\angle EAR$ .

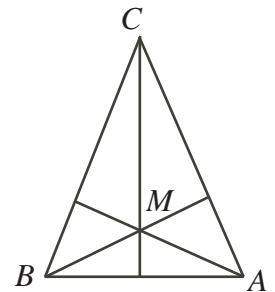




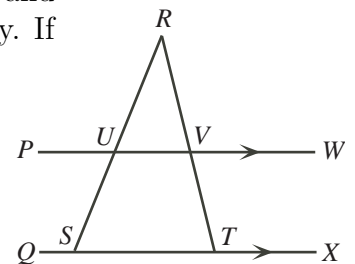
17. A beam of light shines from point  $S$ , reflects off a reflector at point  $P$ , and reaches point  $T$  so that  $PT$  is perpendicular to  $RS$ . What is the value of  $x$ ?



18. In the diagram, let  $M$  be the point of intersection of the three altitudes of triangle  $ABC$ . If  $AB = CM$ , then what is  $\angle BCA$  in degrees?



19. In the diagram,  $PW$  is parallel to  $QX$ ,  $S$  and  $T$  lie on  $QX$ , and  $U$  and  $V$  are the points of intersection of  $PW$  with  $SR$  and  $TR$ , respectively. If  $\angle SUV = 120^\circ$  and  $\angle VTX = 112^\circ$ , what is the measure of  $\angle URV$ ?





20. Three regular polygons meet at a point and do not overlap. One has 3 sides and one has 42 sides. How many sides does the third polygon have? Can you find other sets of three polygons that have this property?

Answers

- |                                  |                                |                              |
|----------------------------------|--------------------------------|------------------------------|
| 1. $x = 10^\circ, y = 150^\circ$ | 3. $\angle DBC = 28^\circ$     | 4. $x = 120^\circ$           |
| 5. $\angle JFI = 27^\circ$       | 6. $\angle BED = 45^\circ$     | 7. $y = 2x$                  |
| 8. $\angle QPR = 54^\circ$       | 9. $60^\circ$                  | 10. $\angle LOM = 124^\circ$ |
| 11. $x - y + 2z = 180^\circ$     | 12. $136^\circ$ to $144^\circ$ | 13. $145^\circ$              |
| 14. $180^\circ$                  | 16. $\angle EAR = 39^\circ$    | 17. $x = 32^\circ$           |
| 18. $\angle BCA = 45^\circ$      | 19. $\angle URV = 52^\circ$    | 20. 7 sides                  |