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CENTRE FOR EDUCATION IN
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Grade 7/8 Math Circles
Math Contest Solutions
FALL 2012

1. There are 6 balls in the box. 5 of the balls in the box are not grey. Therefore, the probability of selecting a ball that is not grey is $\frac{5}{6}$.

Answer: (E)

2. There are six odd-numbered rows (rows 1, 3, 5, 7, 9, 11).
These rows have $6 \times 15 = 90$ seats in total.
There are five even-numbered rows (rows 2, 4, 6, 8, 10).
These rows have $5 \times 16 = 80$ seats in total.
Therefore, there are $90 + 80 = 170$ seats in total in the theatre.

Answer: (D)

3. 20 is not a prime number, since it is divisible by 2.
21 is not a prime number, since it is divisible by 3.
25 is not a prime number, since it is divisible by 5.
27 is not a prime number, since it is divisible by 3.
23 is a prime number, since its only positive divisors are 1 and 23.

Answer: (C)

4. Kayla walked 8 km on Monday.
On Tuesday, she walked $8 \div 2 = 4$ km
On Wednesday, she walked $4 \div 2 = 2$ km
On Thursday, she walked $2 \div 2 = 1$ km
On Friday, she walked $1 \div 2 = 0.5$ km

Answer: (E)

5. Since Max sold 41 glasses of lemonade on Saturday and 53 on Sunday, he sold $41 + 53 = 94$ glasses in total. Since he charged 25 cents for each glass, then his total sales were $94 \times \$0.25 = \$23.50$.

Answer: (A)

6. Since a large box costs \$3 more than a small box and a large box and a small box together cost \$15, then replacing the large box with a small box would save \$3. This tells us that two small boxes together cost \$12. Therefore, one small box costs \$6.

Answer: (D)

7. In total, there are $2 + 5 + 4 = 11$ balls in the bag. Since there are 5 yellow balls, then the probability of choosing a yellow ball is $\frac{5}{11}$.

Answer: (B)

8. Since each number in the Fibonacci sequence, beginning with the 2, is the sum of the two previous numbers, then the sequence continues as 1, 1, 2, 3, 5, 8, 13, 21. Thus, 21 appears in the sequence.

Answer: (B)

9. *Solution 1*
We look at each of the choices and try to make them using only 3 cent and 5 cent stamps:

   (a) 7 cannot be made, since no more than one 5 cent and two 3 cent stamps could be used
   (b) $13 = 5 + 5 + 3$
   (c) 4 cannot be the answer since a larger number (7) already cannot be made
   (d) $8 = 5 + 3$
   (e) $9 = 3 + 3 + 3$

   Therefore, the answer must be 7.

   Solution 2
   We make a table to determine which small positive integers can be made using 3s and 5s:

Integer	Combination
1	Cannot be made
2	Cannot be made
3	3
4	Cannot be made
5	5
6	3 + 3
7	Cannot be made
8	5 + 3
9	3 + 3 + 3
10	5 + 5
11	5 + 3 + 3

   Every integer larger than 11 can also be made because the last three integers in our table can be made and we can add a 3 to our combinations for 9, 10 and 11 to gets combinations for 12, 13 and 14, and so on. From the table, the largest amount of postage that cannot be made is 7.

Answer: (A)

10. We list all of the possible orders of finish, using H, R and N to stand for Harry, Ron and Neville. The possible orders are HNR, HRN, NHR, NRH, RHN, RNH. Therefore there are 6 possible orders.

Answer: (B)

11. Since x and z are positive integers and $xz = 3$, the only possibilities are $x = 1$ and $z = 3$ or $x = 3$ and $z = 1$.
Assuming that $x = 1$ and $z = 3$, $yz = 6$ implies $3y = 6$ or $y = 2$.
Thus, $x = 1$ and $y = 2$ and $xy = 2$.
This contradicts that first equation $xy = 18$.
Therefore, our assumption was incorrect and it must be true that $x = 3$ and $z = 1$.
Then $yz = 6$ and $z = 1$ implies $y = 6$.
Checking, $x = 3$ and $y = 6$ also satisfies $xy = 18$, the first equation.
Therefore, the required sum is $x + y + z = 3 + 6 + 1 = 10$.

Answer: (B)

12. Since the bamboo plant grows at a rate of 105 cm per day and there are 7 days from May 1st and May 8th, then it grows $7 \times 105 = 735$ cm in this time period. Since 735 cm = 7.35 m, then the height of the plant on May 8th is $2 + 7.35 = 9.35$ m.

Answer: (E)

13. *Solution 1*
Since the sum of the numbers on opposite faces on a die is 7, then 1 and 6 are on opposite faces, 2 and 5 are on opposite faces, and 3 and 4 are on opposite faces. On the first die, the numbers on the unseen faces opposite the 6, 2 and 3 are 1, 5 and 4, respectively. On the second die, the numbers on the unseen faces opposite the 1, 4 and 5 are 6, 2 and 3, respectively. The sum of the missing numbers is $1 + 5 + 4 + 6 + 2 + 3 = 21$.

Solution 2
The sum of the numbers on a die is $1 + 2 + 3 + 4 + 5 + 6 = 21$ and so the sum of the numbers on two die is $2 \times 21 = 42$. Since there is a sum of 21 showing on the six visible faces, the sum of the numbers on the six unseen faces is $42 - 21 = 21$.

Answer: (C)

14. *Solution 1*
Starting at the “K” there are two possible paths that can be taken. At each “A”, there are again two possible paths that can be taken. Similarly, at each “R” there are two possible paths that can be taken. Therefore, the total number of paths is $2 \times 2 \times 2 = 8$. (We can check this by actually tracing out the paths.)

Solution 2
Each path from the K at the top to one of the L’s at the bottom has to spell KARL. There is 1 path

that ends at the first L from the left. This path passes through the first A and the first R. There are 3 paths that end at the second L. The first of these passes through the first A and the first R. The second of these passes through the first A and the second R. The third of these passes through the second A and the second R. There are 3 paths that end at the third L. The first of these passes through the first A and the second R. The second of these passes through the second A and the second R. The third of these passes through the second A and the third R. There is 1 path that ends at the last L. This path passes through the last A and the last R. So the total number of paths to get to the bottom row is 1 + 3 + 3 + 1 = 8, which is the number of paths that can spell KARL.

Answer: (D)

15. First, we try to figure out what digit Q is. Since the product is not equal to 0, Q cannot be 0. Since the product has four digits and the top number has three digits, then Q (which is multiplying the top number) must be bigger than 1. Looking at the units digits in the product, we see that $Q \times Q$ has a units digit of Q. Since $Q>1$, then Q must equal 5 or 6 (no other digit gives itself as a units digit when multiplied by itself). But Q cannot be equal to 5, since if it was, the product $RQ5Q$ would end "55" and each of the two parts (P P Q and Q) of the product would end with a 5. This would mean that each of the parts of the product was divisible by 5, so the product should be divisible by $5 \times 5 = 25$. But a number ending in 55 is not divisible by 25. Therefore, Q = 6. So the product now looks like

$$\begin{array}{r} \text{P P 6} \\ \times \quad\quad 6 \\ \hline \text{R 6 5 6} \end{array}$$

Now when we start the long multiplication, 6×6 gives 36, so we write down 6 and carry a 3. When we multiply $P \times 6$ and add the carry of 3, we get a units digit of 5, so the units digit of $P \times 6$ should be 2. For this to be the case, $P = 2$ or $P = 7$. We can now try these possibilities: $226 \times 6 = 1356$ and $776 \times 6 = 4656$. Only the second ends "656" like the product should. So $P = 7$ and $R = 4$, and so $P + Q + R = 7 + 6 + 4 = 17$.

Answer: (E)

16. *Solution 1*
We want to combine 48 coins to get 100 cents. Since the combined value of the coins is a multiple of 5, as is the value of a combination of nickels, dimes and quarters, then the value of the pennies must also be a multiple of 5.

Therefore, the possible numbers of pennies are 5, 10, 15, 20, 25, 30, 35, 40. We can also see that because there are 48 coins in total, it is not possible to have anything other than 35, 40 or 45 pennies. (For example, if we had 30 pennies, we would have 18 other coins which are worth at least 5 cents each, so we would have at least $30 + 5 \times 18 = 120$ cents in total, which is not possible. We can make a similar argument for 5, 10, 15, 20 and 25 pennies.)

It is also not possible to have 3 or 4 quarters. If we did have 3 or 4 quarters, then the remaining 45 or 44 coins would give us a total value of at least 44 cents, so the total value would be greater than

100 cents. Therefore, we only need to consider 0, 1 or 2 quarters.

Possibility 1: 2 quarters
If we have 2 quarters, this means we have 46 coins with a value of 50 cents. The only possibility for these coins is 45 pennies and 1 nickel.

Possibility 2: 1 quarter
If we have 1 quarter, this means we have 47 coins with a value of 75 cents. The only possibility for these coins is 40 pennies and 7 nickels.

Possibility 3: 0 quarters
If we have 0 quarters, this means we have 48 coins with a value of 100 cents. If we had 35 pennies, we would have to have 13 nickels. If we had 40 pennies, we would have to have 4 dimes and 4 nickels. It is not possible to have 45 pennies.

Therefore, there are 4 possible combinations.

Solution 2
We want to use 48 coins to total 100 cents.
Let us focus on the number of pennies.
Since any combination of nickels, dimes and quarters always is worth a number of cents which is divisible by 5, then the number of pennies in each combination must be divisible by 5, since the total value of each combination is 100 cents, which is divisible by 5.

Could there be 5 pennies? If so, then the remaining 43 coins are worth 95 cents. But each of the remaining coins is worth at least 5 cents, so these 43 coins are worth at least $5 \times 43 = 215$ cents, which is impossible. So there cannot be 5 pennies.
Could there be 10 pennies? If so, then the remaining 38 coins are worth 90 cents. But each of the remaining coins is worth at least 5 cents, so these 38 coins are worth at least $5 \times 38 = 190$ cents, which is impossible. So there cannot be 10 pennies. We can continue in this way to show that there cannot be 15, 20, 25, or 30 pennies. Therefore, there could only be 35, 40 or 45 pennies.

If there are 35 pennies, then the remaining 13 coins are worth 65 cents. Since each of the remaining coins is worth at least 5 cents, this is possible only if each of the 13 coins is a nickel. So one combination that works is 35 pennies and 13 nickels.

If there are 40 pennies, then the remaining 8 coins are worth 60 cents. We now look at the number of quarters in this combination. If there are 0 quarters, then we must have 8 nickels and dimes totalling 60 cents. If all of the 8 coins were nickels, they would be worth 40 cents, so we need to change 4 nickels to dimes to increase our total by 20 cents to 60 cents. Therefore, 40 pennies, 0 quarters, 4 nickels and 4 dimes works.

If there is 1 quarter, then we must have 7 nickels and dimes totalling 35 cents. Since each remaining coin is worth at least 5 cents, then all of the 7 remaining coins must be nickels. Therefore, 40 pennies, 1 quarter, 7 nickels and 0 dimes works. If there are 2 quarters, then we must have 6 nickels and dimes totalling 10 cents. This is impossible. If there were more than 2 quarters, the quarters would be worth more than 60 cents, so this is not possible.

If there are 45 pennies, then the remaining 3 coins are worth 55 cents in total. In order for this to be possible, there must be 2 quarters (otherwise the maximum value of the 3 coins would be with 1 quarter and 2 dimes, or 45 cents). This means that the remaining coin is worth 5 cents, and so is a nickel. Therefore, 45 pennies, 2 quarters, 1 nickel and 0 dimes is a combination that works.

Therefore, there are 4 combinations that work.

Answer: (B)