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Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles
Sequences and Series

PROBLEM SET SOLUTIONS

1. Description: Closed form is $3n$. Recursive is $t_n = t_{n-1} + 3$ with $t_1 = 3$. Arithmetic is $t_n = 3 + (n-1)(3)$ The 10th term is 30.

2. Description: Closed form is $8n - 5$. Recursive is $t_n = t_{n-1} + 8$ with $t_1 = 3$. Arithmetic is $t_n = 3 + (n-1)(8)$. The 8th term is 59.

3. Description: Closed form is $23 - 2n$. Recursive form is $t_n = t_{n-1} - 2$ with $t_1 = 25$. Arithmetic is $t_n = 25 + (n-1)(-2)$. The 11th term is 45.

4. Description: Closed form is $120 - 5n$. Recursive form is $t_n = t_{n-1} - 5$ with $t_1 = 125$. Arithmetic is $t_n = 25 + (n-1)(-5)$. The 11th term is 65.

5. Description: Closed form is $2n - 27$. Recursive form is $t_n = t_{n-1} + 2$ with $t_1 = -25$. Arithmetic is $t_n = -25 + (n-1)(2)$. The 11th term is 15.

6. Closed form is $t_n = 4^{n-1}$. Recursive form is $t_n = 4 \times t_{n-1}$ with $t_1 = 1$. There is no arithmetic formula. The 6th term is 1024.

7. Closed form is $t_n = 2^n - 1$. Recursive form is $t_n = 2 \times t_{n-1} + 1$ with $t_1 = 1$. This is not an arithmetic sequence so it has no arithmetic formula. The 13th term is 8191.

8. Using logic, and following the pattern, we see that the 24th digit will be a 2. Then the next digit (25th) is a 0, and the 26th digit then has to be a 1.

9. Using logic, we see that it will be a 5.

10. Using our formula that we derived in class, (123 - 1) $\div$ 2 + 1 = 62 terms.

11. Using our formula that we derived in class, (121 - 1) $\div$ 4 + 1 = 31 terms.

12. The pattern is that each time you add a side, you increase the number of degrees by 180. This helps us find the arithmetic sequence formula $t_n = 180 + (n-1)(180)$, where t_1 is what we want for 3 sides. Then to get the 12-sides, we

want t_10. So plugging in n = 10, we get 1800. A regular dodecagon means that all the angles are equal - so since there are 12 interior angles (12 sides = 12 corners), we divide by 12 to get 150.

13. We notice that the sequence for the rule n^3 is $\{1, 8, 27, 64, 125, 216, 343, 512, 729, \dots\}$. We notice that the 1st term in the n^3 sequence, minus 1, is 0, the first term in the question's sequence. The 2nd term, minus 2, is 6, which is the second term in the question's sequence. 3rd term, minus 3, works. 4th term, minus 4, works. So each time, for the n^{th} term in the sequence, we are finding n^3, then subtracting n! Therefore, the rule is $n^3 - n$. To find the 11th, just plug in 11 for n, and you'll get 1320.

14. If we actually evaluate the terms in the sequence with a calculator, we have the sequence $\{7, 49, 343, 2401, 16807, \dots\}$. It's obviously a pain to do the multiplication, but you should have learned before, that when you multiply two numbers, you can find the ones digit of the product just by multiplying the ones digits of the two numbers! For example, the ones digit of 127891242 and 133143141 is just 2, because the $2 \times 1 = 2$!.

    Then we can easily find the sequence of the ones digits of each of these numbers.

    The first term in this sequence is 7, since the first term in the original sequence is 7. To get the next digit, multiply 7 by 7, which is 49, and then take the ones digit, which is 9. For the third term, multiply 9 by 7, which is 63; take the ones digit, and it is 3! Continue this pattern and you will notice that you get the sequence $\{7, 3, 9, 1, 7, 3, 9, 1, \dots\}$. This looks a lot like question 8, doesn't it? I'll let you figure out the details on how to find the 63rd term, but the answer is 9. The ones digit of the 63rd term in this sequence, is a 9.