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Centre for Education in
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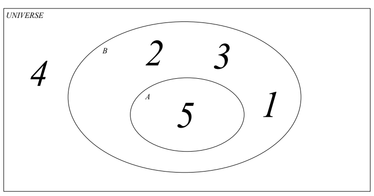
# Grade 7/8 Math Circles
# Sequences and Series

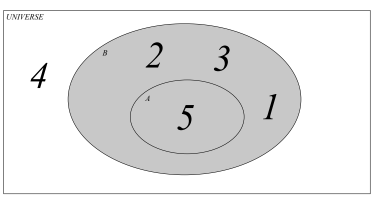
PROBLEM SET SOLUTIONS

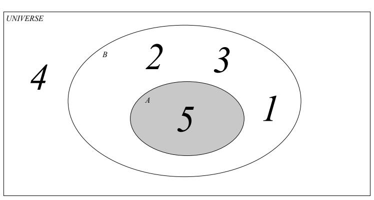
1. This is not a valid set; we don't allow duplicates.

2. Yes, since 6 is even.

3. No, containment means all or nothing.

4. (a) 


   (b) 


   (c) 


   (d) $A^c = \{1, 2, 3, 4\}$, $B^c = \{4\}$

   (e) $(A^c)^c = \{5\} = A$, $(B^c)^c = \{1, 2, 3, 5\} = B$

   (f) Taking the complement twice just gives you the set X back.

5. (a) Yes it is. The statement reads from left to right.

   (b) Yes it is.

6. It is the empty set. When you combine two sets with nothing, you will get a set that has nothing.

7. It is the empty set. The intersection is the set containing elements that are common to both sets. Nothing is common to both sets. So the intersection must be the empty set. (This also follows from our definition that any set intersect the empty set is the same set).

8. $\mathbb{U}$ = {Euclid, Riemann, Hilbert, Newton, Galois, Gauss, Abel, Godel, Wiles, Chern}

   (a) $R^c$ = {Galois, Gauss, Abel, Godel, Wiles, Chern}

   (b) $(S \cap T)^c = \mathbb{U}$, same for $S^c \cup T^c$.

9. $\phi$ since $S \cap T = \phi$

10. First blank is R, second blank is ∪, third blank is ∩.

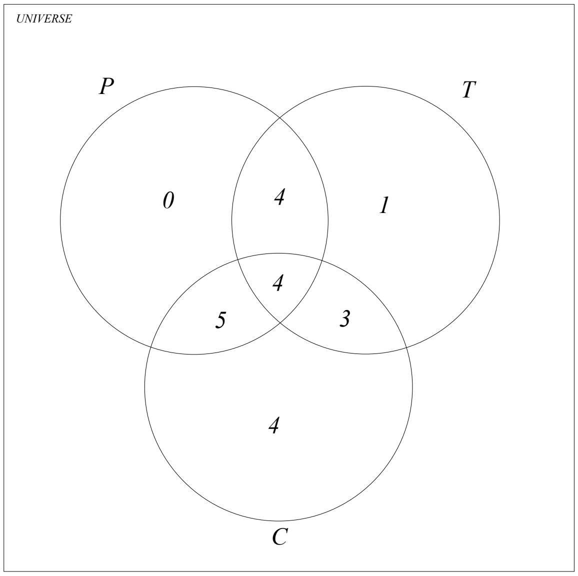
11. 24 + 12 - 7 = 29.

12. Let S = {safecrackers}, G = {getaway drivers}, and let $x = \#(S \cap G)$ - this is what we are looking for. Let's use the formula for the Inclusion Exclusion Principle.

    From the question, we know that #S = 17, #G = 28, and #(S ∪ G) = 40.

$$
\begin{aligned}
\#S + \#G - \#(S \cap G) &= \#(S \cup G) \\
17 + 28 - \#(S \cap G) &= 40 \\
45 - \#(S \cap G) &= 40 \\
\therefore \#(S \cap G) &= 5
\end{aligned}
$$

    So there are 5 people in the group.

13. 


Therefore no students are enrolled only in Potions.

14. The Inclusion Exclusion Principle for 3 sets P, T, C, is

$$\# P + \# T + \# C - \#(P \cap T) - \#(P \cap C) - \#(T \cap C) + \#(P \cap T \cap C) = \#(P \cup T \cup C)$$

The first three terms happen when we add up the sizes of the three sets. (This is #P + #T + #C).

When we do this, though, we have to remember that any pair of these sets overlap with each other. So we have to subtract all the times they overlap once, to eliminate double counting. This is why we subtract $\#(P \cap T)$, $\#(P \cap C)$, and $\#(T \cap C)$.

However, don't forget the portion that is in all three overlaps. When we added up #P + #T + #C, this part was TRIPLE counted (there were TWO overlaps on it).

When we subtracted $\#(P \cap T)$, this eliminated one overlap. When we subtracted $\#(P \cap C)$, this eliminated another overlap, and we should have stopped here, because we eliminated the two overlaps.

However, we had to also subtract $\#(T \cap C)$ - this means we subtracted one too many times. Therefore, we had to add back this amount we subtracted, which is why we add $\#(P \cap T \cap C)$.