Math Circles April 3, 2013

Exercises About Series

1. Find a simple expression for the nth partial sum of the series

$$\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right)$$

and hence find its sum. *Hint: This type of series is referred to as* telescoping.

2. Prove that the real number $0.999999... = 0.\bar{9}$ (i.e. the terms continue forever, with all digits equal to 9) is equal to one. *Hint: Write the number as 0.9 + 0.09 + 0.009 + ...*

3. Write the number $2.3\overline{17} = 2.3171717...$ as a ratio of integers.

4. Is the series $\sum_{n=1}^{\infty} 2^{2n} 3^{1-n}$ convergent or divergent? If it is convergent, determine the sum.

5. Is the series $\sum_{n=1}^{\infty} 2^{2n} 5^{1-n}$ convergent or divergent? If it is convergent, determine the sum.

6. When light hits a certain pane of glass, the glass reflects one half of the light, absorbs one fourth, and transmits one fourth. A window is made of two panes of this glass separated by a small gap. If light of intensity I shines directly onto the window, what fraction is transmitted to the other side of the double pane?