

# Math Circles: Contest Preparation II Full Solution Contests

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February 13, 2013

# Agenda

- 1 Review of Multiple Choice Contests
- 2 Review of Some of the Strategies
- 3 Warm-Up
- 4 Working on Full Solution Problems
- 5 Some Concluding Thoughts

Pascal  
Cayley  
Fermat

Thursday, February 21, 2013

# Contest Hints

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# Remember the eWorkshop?

[Pascal, Cayley, Fermat eWworkshop](#)

## Warm - Up    Not Your Ordinary Magic Square

A 4 by 4 “anti-magic” square is an arrangement of the numbers 1 to 16 inclusive in a square so that the totals of each of the four rows and four columns and two main diagonals are ten consecutive numbers in some order. The diagram shows an incomplete “anti-magic” square. When the chart is completed what number will replace  $\alpha$ ?

4	5	7	14
6	13	3	$\alpha$
11	12	9	
10			

Thursday, April 18, 2013

**Fryer**

**Galois**

**Hypatia**

November 2013

**Canadian Intermediate  
Math Contest**


# Full Solution Contest Information

1. Light Bulb Questions / Answer in Box Questions

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From Fryer, Galois, Hypatia

1. **SHORT ANSWER** parts indicated by 


- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided



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- worth 2 or 3 marks each
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From CIMC

### **PART A**

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**  
For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

# Full Solution Contest Information

## 2. Full Written Solutions

# Full Solution Contest Information

## 2. Full Written Solutions

From Fryer, Galois, Hypatia

2. **FULL SOLUTION** parts indicated by



- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks

## 2. Full Written Solutions

From Fryer, Galois, Hypatia

2. **FULL SOLUTION** parts indicated by



- worth the remainder of the 10 marks for the question
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- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks

From CIMC



### **PART B**

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

## 3. TIPS

## 3. TIPS

From Fryer, Galois, Hypatia

- TIPS:
1. Please read the instructions on the front cover of this booklet.
  2. Write all answers in the answer booklet provided.
  3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
  4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
  5. Diagrams are *not* drawn to scale. They are intended as aids only.

## 3. TIPS

## 3. TIPS

### From CIMC

- |       |    |  |
|-------|----|--|
| NOTE: | 1. | Please read the instructions on the front cover of this booklet.   |
|       | 2. | Write solutions in the answer booklet provided.  |
|       | 3. | It is expected that all calculations and answers will be expressed as exact numbers such as $4\pi$ , $2 + \sqrt{7}$ , etc., rather than as $12.566\dots$ or $4.646\dots$ |
|       | 4. | <b>Calculators are permitted</b> , provided they are non-programmable and without graphic displays.  |
|       | 5. | Diagrams are not drawn to scale. They are intended as aids only.   |

#### NOTES:

The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.



# Full Solution Contests

Go to [www.cemc.uwaterloo.ca](http://www.cemc.uwaterloo.ca) for old contests.

From Fryer, Galois, Hypatia

Fryer (Grade 9)										<a href="#">back to top</a>
<b>Contests</b>	<a href="#">2012</a>	<a href="#">2011</a>	<a href="#">2010</a>	<a href="#">2009</a>	<a href="#">2008</a>	<a href="#">2007</a>	<a href="#">2006</a>	<a href="#">2005</a>	<a href="#">2004</a>	
	<a href="#">2003</a>	<a href="#">Download all as single PDF</a>								
<b>Solutions</b>	<a href="#">2012</a>	<a href="#">2011</a>	<a href="#">2010</a>	<a href="#">2009</a>	<a href="#">2008</a>	<a href="#">2007</a>	<a href="#">2006</a>	<a href="#">2005</a>	<a href="#">2004</a>	
	<a href="#">2003</a>									
<b>Results</b>	<a href="#">2012</a>	<a href="#">2011</a>	<a href="#">2010</a>	<a href="#">2009</a>	<a href="#">2008</a>	<a href="#">2007</a>	<a href="#">2006</a>	<a href="#">2005</a>	<a href="#">2004</a>	
	<a href="#">2003</a>									
Galois (Grade 10)										
<b>Contests</b>	<a href="#">2012</a>	<a href="#">2011</a>	<a href="#">2010</a>	<a href="#">2009</a>	<a href="#">2008</a>	<a href="#">2007</a>	<a href="#">2006</a>	<a href="#">2005</a>	<a href="#">2004</a>	
	<a href="#">2003</a>	<a href="#">Download all as single PDF</a>								
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	<a href="#">2003</a>									
Hypatia (Grade 11)										
<b>Contests</b>	<a href="#">2012</a>	<a href="#">2011</a>	<a href="#">2010</a>	<a href="#">2009</a>	<a href="#">2008</a>	<a href="#">2007</a>	<a href="#">2006</a>	<a href="#">2005</a>	<a href="#">2004</a>	
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# Full Solution Contests

Go to [www.cemc.uwaterloo.ca](http://www.cemc.uwaterloo.ca) for old contests.

From CIMC / CSMC

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Team Problems	<a href="#">2012</a>	
Answer Key	<a href="#">2012</a>	

# Full Solution Contests

Tonights problems have been taken mainly from past CEMC Contests.

## Problem #1

Four years ago, Daryl was three times as old as Joe was. In five years, Daryl will be twice as old as Joe will be. How old is Daryl now?

## Problem #2

Determine the number of positive divisors of 18 800 that are divisible by 235.

## Problem #3

What is the area of  $\triangle ABC$  with vertices  $A(3, -1)$ ,  $B(5, 1)$  and  $C(8, 7)$ ?

## Problem #4

The average of three consecutive multiples of 3 is  $a$ . The average of four consecutive multiples of 4 is  $(a + 27)$ . The average of the smallest and largest of these seven integers is 2022. Determine the value of  $a$ .

## Problem #5

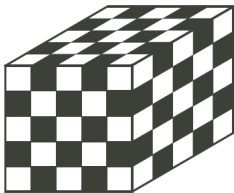
At last week's hockey game involving the Waterloo Blueberries, the price of a platinum ticket was \$25, the price of a gold ticket was \$10, the price of a silver ticket was \$5, and the price of a bronze ticket was \$1.

- a) Jim buys 5 platinum tickets, 2 gold tickets and 3 silver tickets. Determine the average cost of the tickets that Jim buys. Explain how you obtained your answer.
- b) Mike buys 8 tickets whose average cost is \$12. He then buys 5 more platinum tickets. What is the new average cost of the tickets that he has bought? Explain how you obtained your answer.
- c) Ophelia buys 10 tickets with an average cost of \$14. Suppose that she buys  $n$  more platinum tickets. The new average cost of the tickets that she has bought is \$20. What is the value of  $n$ ? Explain how you obtained your answer.



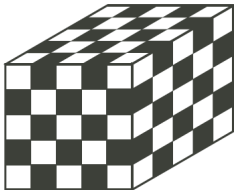
## Problem #6

- a) Each face of a 5 by 5 by 5 wooden cube is divided into 1 by 1 squares. Each square is painted black or white, as shown. Next, the cube is cut into 1 by 1 by 1 cubes. How many of these cubes have at least two painted faces?



## Problem #6

- b) A  $(2k + 1)$  by  $(2k + 1)$  by  $(2k + 1)$  cube, where  $k$  is a positive integer, is painted in the same manner as the 5 by 5 by 5 cube with white squares in the corners. Again, the cube is cut into 1 by 1 by 1 cubes.
- In terms of  $k$ , how many of these cubes have exactly two white faces?
  - Prove that there is no value of  $k$  for which the number of cubes having at least two white faces is 2006.



## Problem #7

The number 8 is the sum and product of the numbers in the collection of four positive integers  $\{1, 1, 2, 4\}$ , since  $1 + 1 + 2 + 4 = 8$  and  $1 \times 1 \times 2 \times 4 = 8$ . The number 2007 can be made up from a collection of  $n$  positive integers that multiply to 2007 and add to 2007. What is the smallest value of  $n$  with  $n > 1$ ?