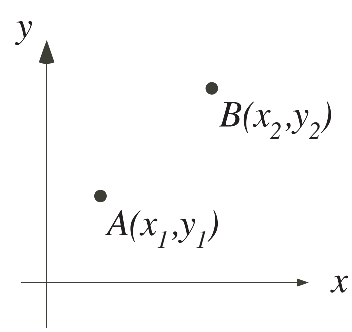
Intermediate Math Circles
February 20, 2013
Analytic Geometry I

1. The Cartesian Plane

We use a coordinate system to allow us to translate a geometric problem into an algebraic problem.

We bring a lot to the table: angle properties and theorems, similar and congruent triangles, etc. In the first four weeks of the fall we examined Euclidean Geometry learning facts about angles, side lengths, and circles.

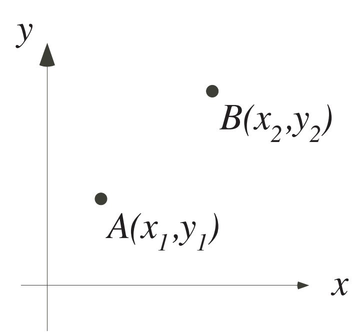
Credit for developments in the area of Analytic Geometry go to Rene Descartes. Descartes is also credited with the phrase: "*I think, therefore I am.*"



The coordinate system requires an origin, an x-axis, and a y-axis with which you should be familiar. Later in math we will extend to a third dimension.

A *point* is a specific location on the Cartesian Plane.

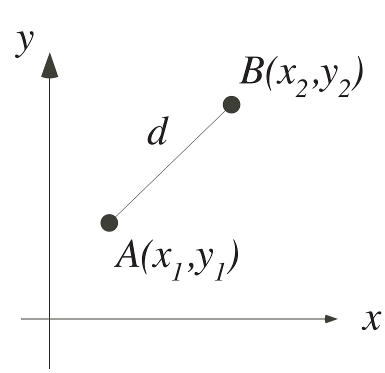
A straight line can be drawn through two points. It has no beginning and no end.



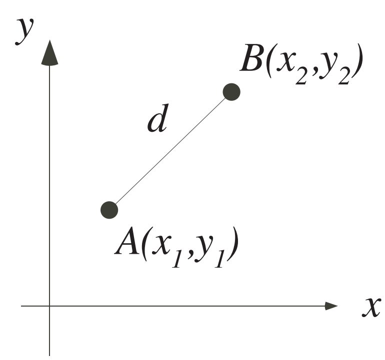
AB is a line segment with endpoints A and B. A line segment is different from a line because it has a fixed length.

2. Distance Between Two Points

If d is the distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ then



2. Distance Between Two Points (continued)



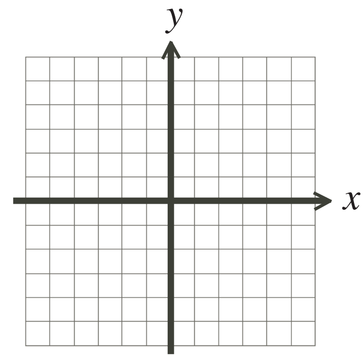
If d is the distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ then $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Proof:

Other Useful Information Concerning Distances

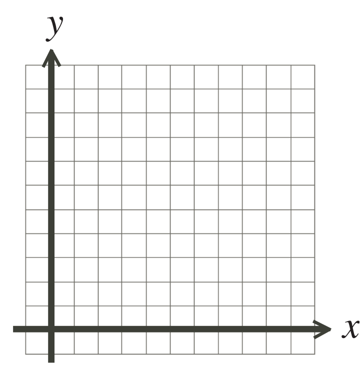
Problem (i)

The line segment joining $A(2,6)$ to $B(8,-2)$ forms the base of isosceles $\triangle ABP$. The x-coordinate of the third vertex P is -5 and $AP = BP$. Determine the y-coordinate of point P.



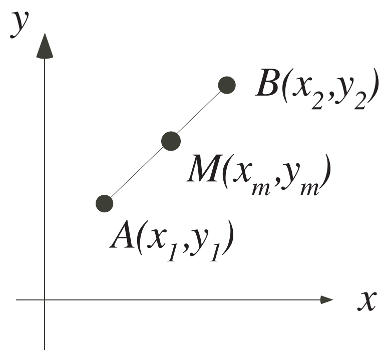
Problem (ii)

Determine the centre of a circle which passes through points $P(0,3)$, $Q(2,-1)$, and $R(9,0)$.



3. Midpoint Between Two Points

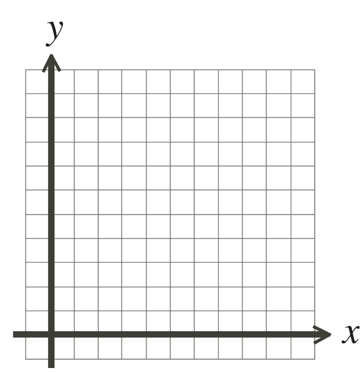
If $M(x_m, y_m)$ is the midpoint of AB, then



Proof:

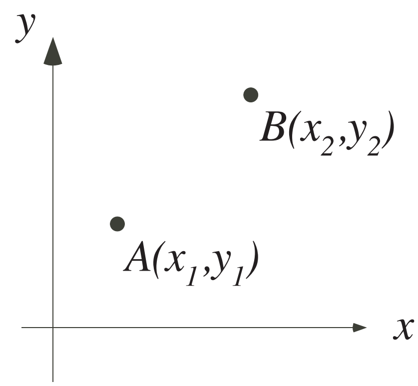
Problem (iii)

Points P, Q, and R divide the line segment from $A(0, 2)$ to $C(6, 0)$, in that order, into four equal parts. Determine the coordinates of the three points, P, Q, and R.



4. Slope of a Line / Line Segment

(a) Definition



Slope is a measure of the steepness of a line (or line segment).

Slope is generally represented by the letter m.

$$m =$$

(b) Special Cases

i) Horizontal Lines

ii) Vertical Lines

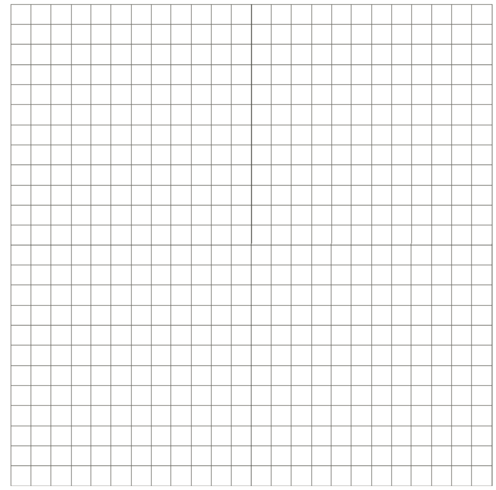
iii) Parallel Lines

iv) Perpendicular Lines

Problem (iv)

$\triangle BAH$ has vertices at $B(-1, 9)$, $A(a, 1)$ and $H(-7, -4)$ such that $\angle BAH = 90^\circ$. Determine the value of a.

Present two different solutions.



Work on the problem set. Full solutions will be posted on the website later this week.