



Intermediate Math Circles

Number Theory II

Problems

1. The difference between the gcd and lcm of the numbers 10, 15, 35 is
(a) 60 (b) 205 (c) 25 (d) 1044 (e) 5245
2. Two speed skaters begin practice at the same start line. The first speed skater completes one lap of the oval every 45 seconds. The second speed skater completes one lap of the oval every 63 seconds. After the start, how much time will elapse until the speed skaters are at the start line together again?
(a) $47\frac{1}{4}$ min (b) $9\frac{9}{20}$ min (c) $5\frac{1}{4}$ min (d) $2\frac{1}{10}$ min (e) $15\frac{3}{4}$ min
3. When a positive integer is divided by 7, the quotient is 4 and the remainder is 6. Determine the value of this number.
4. An uptown bus leaves the terminal every 70 minutes and a downtown bus leaves the same terminal every 42 minutes. If the uptown and downtown busses both leave the terminal at 9:00am, when will they be at the terminal together again?
5. Erin's age when divided by 2, 3, 4, 5 or 6 gives a remainder of 1. Determine the youngest possible age that Erin could be, if she is older than 1.
6. When y is divided by $2x - 4$ the quotient is $3x$ and the remainder is 9. Determine the simplified expression of y .
7. Determine the smallest positive integer that is divisible by both 24 and 30.
8. Determine the smallest value of k that makes the product $48k$ divisible by 36.
9. The student's council of your school will be selling hot dogs at the Winter Fair. Hot dogs are sold in packages of 12, but hot dog buns are sold in packages of 8. If the student's council predicts that they will need at least 150 hot dogs for the Fair, how many packages of hot dogs and buns should they buy, if every hot dog must be paired with a bun?

10. A florist has 648 roses, 288 orchids and 432 tulips to create identical bouquets. What is the largest number of identical bouquets that can be created without having any flowers left over? How many flowers of each type will be in one bouquet?
11. Determine the $\gcd(a, b)$ and $\text{lcm}(a, b)$ for the pairs,
(a) $(10!, 6^8)$ (b) $(130\,339, 9061)$
12. Determine the largest value of k for $360x - 540y = k(ax - by)$ such that a, b , and k are positive integers.
13. Determine the exact value of $-\frac{12}{2431} + \frac{9}{1309}$.