UNIVERSITY OF
WATERLOO
Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Grade 7 & 8 Math Circles
Math Circles Review
April 2/3, 2013

Fibonacci & the Golden Ratio

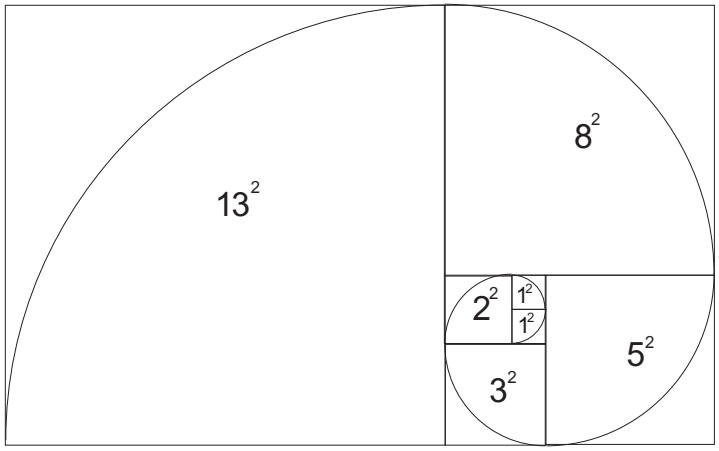
The first few terms (or numbers) in the Fibonacci Sequence are 1, 1, 2, 3, 5, 8,...

To find the next number in the sequence, _____

The formula used to find the nth Fibonacci number is:

$$F_n = \_\_\_\_\_$$

If we draw many squares each with a side length of a Fibonacci number, arrange these squares by increasing area, and connect certain corners with an arc, we get a Fibonacci Spiral. Part of a Fibonacci Spiral is shown below.



Recall that if we take any two consecutive Fibonacci numbers as a fraction, the number will approach _____. This value is called _____ and can be represented by the Greek letter φ (pronounced 'phi')

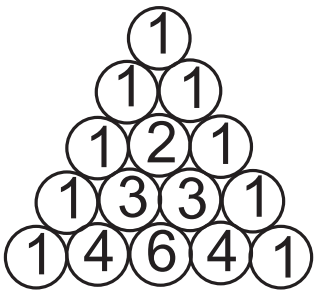
$\varphi = \frac{1+\sqrt{5}}{2} = 1.618034$ can be used to approximate the value of phi.

Using phi, there is another formula we can use to find the nth Fibonacci number:

$$F_n = \frac{\varphi^n - (1-\varphi)^n}{\sqrt{5}}$$

Pascal's Triangle

Here is the first part of Pascal's Triangle.



The number in any circle is the sum of the two numbers in the circles directly above it.

There are so many patterns involved in Pascal's Triangle. We will just review some of them.

Diagonals:

The first diagonal contains only ones.

The second diagonal contains the counting numbers.

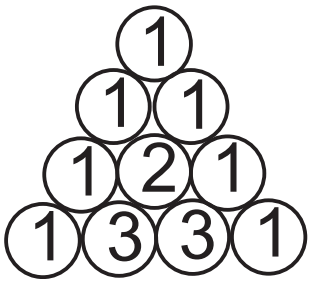
The third diagonal contains all the triangular numbers. Triangular numbers, are the numbers of dots that it takes to make complete triangles. When you look at Pascal's Triangle, the total number of circles required when a new level is added corresponds to a triangular number.

Horizontal Sum: The sum of the numbers in any row is double that of the previous row.

Hockey Sticks: Start at a 1, and draw a hockey stick shape away from the other 1s. The sum of numbers in the long section of the stick will equal the number in the short part of the stick.

Recall that Pascal's Triangle can also be used to calculate combinations. The *choose function* is used to figure out how many different ways a specific number of objects can be chosen from a group. So $\binom{3}{2}$ means how many different combinations are there of choosing 2 objects from a total of 3 objects. The answer corresponds to the third number in the fourth row. Remember we always include 0, so $\binom{0}{0}$ refers to the first number on the first row and $\binom{3}{0}$ refers to the first number on the fourth row.

$$\binom{0}{0}$$
$$\binom{1}{0}\ \binom{1}{1}$$
$$\binom{2}{0}\ \binom{2}{1}\ \binom{2}{2}$$
$$\binom{3}{0}\ \binom{3}{1}\ \binom{3}{2}\ \binom{3}{3}$$



To calculate any *choose* without using Pascal's Triangle we use the formula $\binom{n}{k} =$ where n! means 'n factorial,' or multiply every integer together from n down to 1.

Angles

Here are some important definitions;

Complementary Angles: angles that add up to 90°

Supplementary Angles: angles that add up to 180°

Opposite Angles: when two lines intersect angles directly across the intersection point are equal

Alternate Angles: 'Z' pattern. Angles within the 'Z' are equal

Corresponding Angles: 'F' pattern. Angles in the 'F' are equal

There is a relationship between the angles in a polygon and n, the number of sides it has. The sum of the interior angles in any polygon is _____

3D Geometry

The formula for the volume of any prism is _____

Volume of a cone is $V = \frac{1}{3}\pi r^2 \times$ height

Volume of a pyramid is $V = \frac{1}{3} \times$ area of base $\times$ height

Volume of a sphere is $V = \frac{4}{3}\pi r^3$

Euler's Formula deals with faces, edges and vertices of polyhedrons. Euler found the following relationship was true for all polyhedrons: _____

Graph Theory

Graphs are made up of labelled vertices and edges.

A path is a list of adjacent vertices which never repeats a vertex. A cycle is a path with no vertex repeated except the first and last which must be the same.

The length of a path or cycle is the number of edges in the path or cycle.

A _____ cycle has every vertex of the graph in the cycle.

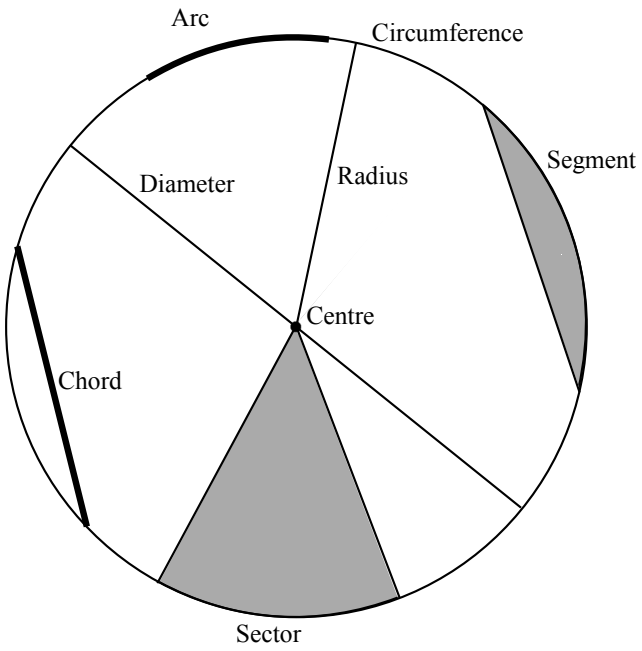
The same graph can be drawn in many different ways. If the vertex labels are the same and all the edges connect the same vertices in two separate graphs, then the graphs are actually the same.

Graphs which are the same except for the vertex labels are said to be isomorphic. If we rename the vertices of one graph to the names of the other and show the edges still connect the same vertices we prove the graphs are isomorphic. We use f(H)= viii to say we've changed the name on vertex H to the name viii.

Trees are connected graphs with no cycles. We can draw the tree so any vertex is the _____, with all other vertices branching out from it. A family tree is a common example of a tree.

Circles, Circles, Circles

The diagram below summarizes all the components of a circle.



Recall two important formulas you should already know about circles:

Area =

Circumference =

A _____ is a long line which touches the circumference of a circle at exactly one point. You can also think of this as extending one of the very short sides of the circle.

To convert an angle from degrees to radians use the following formula: $R = D \times \pi \div 180$

To convert an angle from radians to degrees use the following formula: $D = R \times 180 \div \pi$

The length of an arc is modelled by the equation $L = z \times r$ where z is the angle in **radians**!

The area of a sector is $A = \dfrac{r \times L}{2}$

Remember when we use π we want to leave this symbol in the equations and final answer when possible. When needed, you can use the value 3.14 for π.

When a shape is drawn inside another, we say the shape is _____ in the other.

When 2 equal circles centred at adjacent vertices are drawn and a line connects the intersection points, this line divides the edge in two equal lengths and is perpendicular to the edge. Hence this line is called a _____.

When all these lines are drawn on a polygon and if they intersect at one point, this intersection point is called the _____. This becomes the centre of the circle which inscribes the polygon.

Gauss Contest Preparation

Reminder: The Gauss Contest will be written on **Wednesday, May 15, 2013**.

Below are the three most important tips which are very helpful when solving problems in general, not just when writing the Gauss Contest.

- Draw a Diagram. Remember that diagrams don't need to be drawn to scale.
- Look for patterns. There may be many different patterns you could find.
- Ask yourself, "What important information am I given? What concepts do the writers want to test? What can I do with this information?"

Good luck writing the Gauss Contest and remember to have fun!