UNIVERSITY OF WATERLOO
University of Waterloo
Faculty of Mathematics

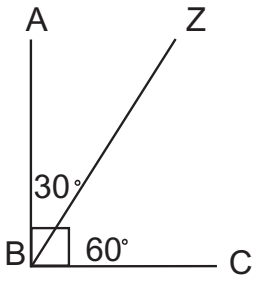
Enriching Mathematics and Computer Science for 50 years

# Grade 7/8 Math Circles
# Winter 2013
# *Angles*

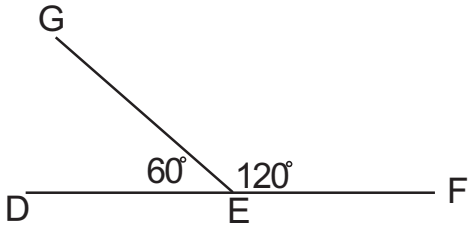
**What is an angle?**

An *angle* is the measure of space within two, or three, or more lines that diverges from a common point.

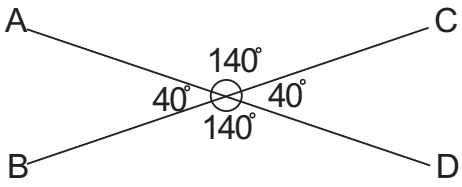
**Complementary Angles**: angles that add up to 90°




**Supplementary Angles**: angles that add up to 180°



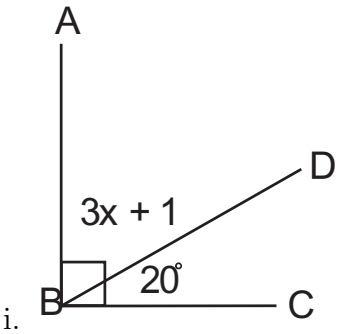

**Opposite Angles**: *equal* angles when two lines intersect.



**Exercise**

Find the value of $x$.

i.

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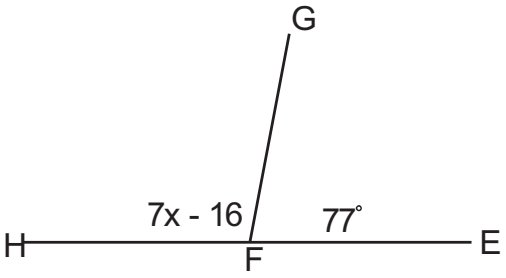
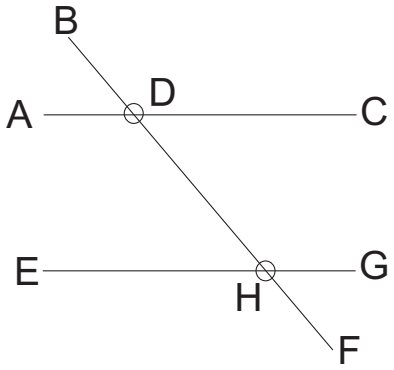


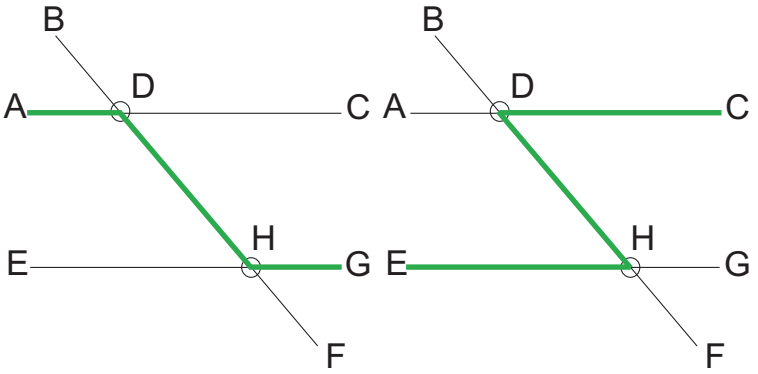

**Traversal**: is a line that intersects two or more parallel lines.

From traversal lines we get *alternate* angle and *corresponding* angles.




All pairs of alternate angles are equal to each other, and all pairs of corresponding are also equal to each other.

To easily see **alternate** angles, draw the letter "Z" on the traversal.

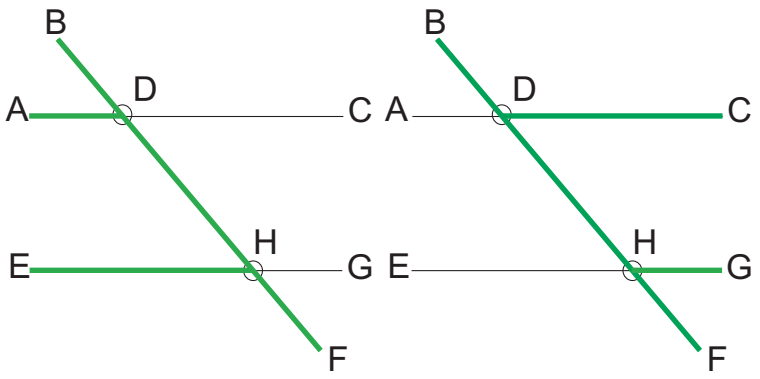


All the angles on the Z are alternate angles.

On the left the alternate angles are $\angle ADH$ and $\angle DHG$.

On the right the alternate angles are $\angle CDH$ and $\angle DHE$.

To easily see **corresponding** angles, draw the letter "F" on the traversal.




All the angles on the F are corresponding angles.

On the left the corresponding angles are $\angle CDH$ and $\angle GHF$, and $\angle BDC$ and $\angle DHG$.

On the right the corresponding angles are $\angle ADH$ and $\angle EHF$, and $\angle ADB$ and $\angle EHD$.

**Exercise**

1) What other angles, of any kind, can you find in the traversal?

   Solution:

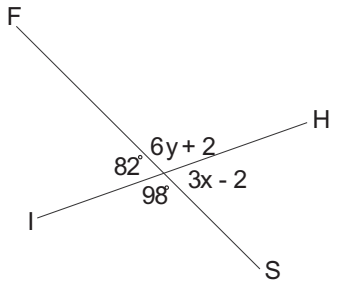
   You have supplementary angles: $\angle ABD$ and $\angle BDC$, $\angle ADH$ and $\angle CDH$, $\angle EHD$ and $\angle DHG$, $\angle EHF$ and $\angle GHF$.

   There are opposite angles: $\angle ADB$ and $\angle CDH$, $\angle ADH$ and $\angle BDC$, $\angle EHD$ and $\angle GHF$, $\angle EHF$ and $\angle DHG$.

   There are more alternate angles: $\angle BDC$ and $\angle DHE$, $\angle ADB$ and $\angle GHF$.

2) Find all the missing values and angles.

i.

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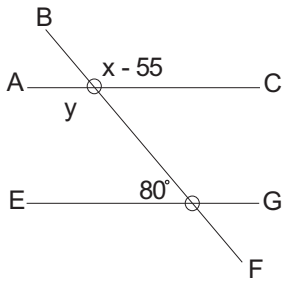


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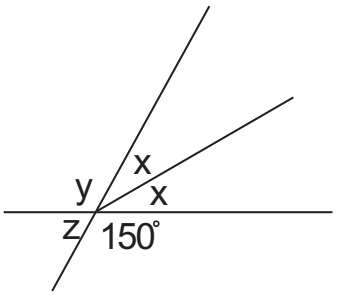


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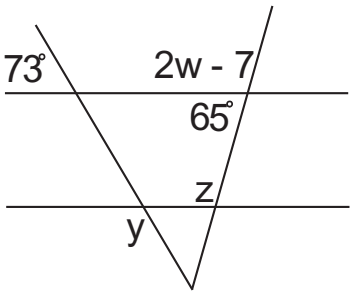
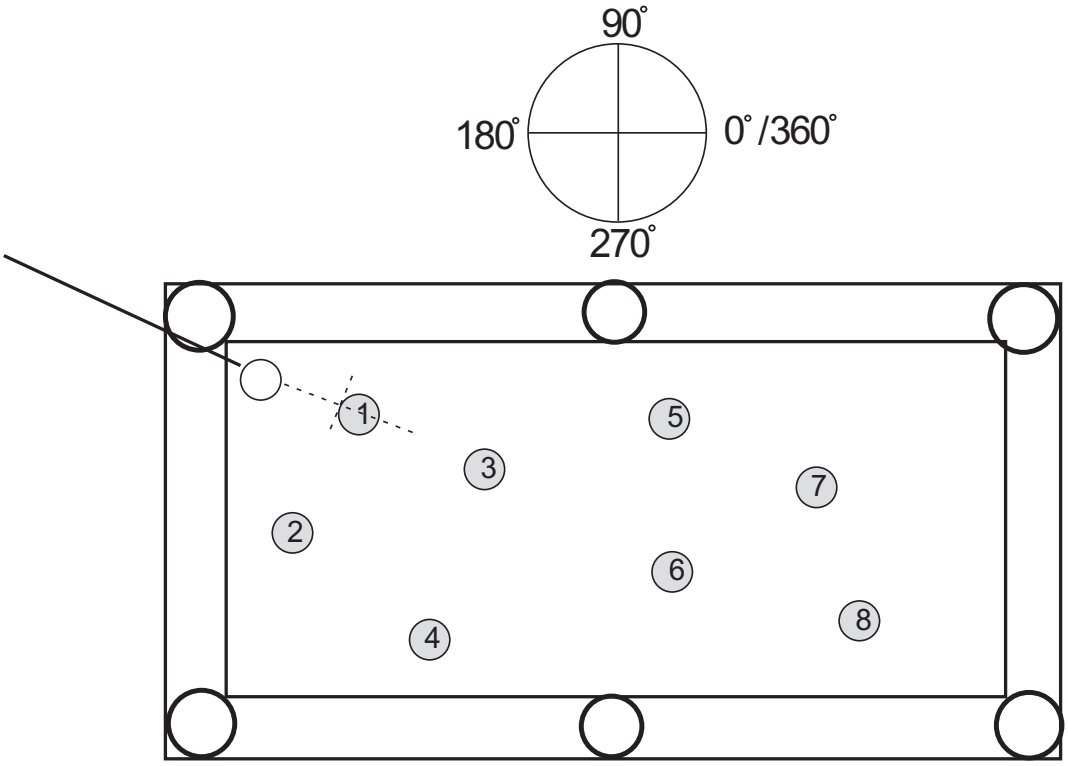



**Pool Table Exercise**

The goal of pool is to shoot the billiard balls on the table into the pockets of the pool table, using a stick, called the cue, and white ball, called the cue ball. Angles are formed when the cue hits the cue ball, which then hits the other billiard balls on the table, or when the cue ball, or any of the other billiard balls, hit the side of the table.

Below is a pool table with billiard balls positioned through out. To start, estimate the angle that is needed for the cue ball to hit ball 1. The cue ball will then be in the position of ball 1. Estimate the following angles needed for the cue ball in position of ball 1, to hit ball 2, etc. The goal is to hit ball 8 into a pocket.

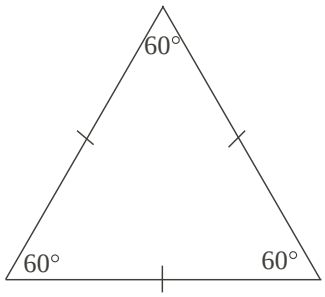
1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____



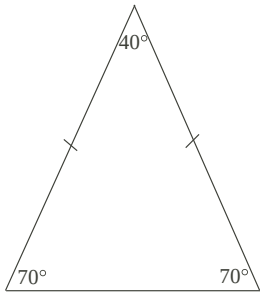

**Angles in Triangles**

There are 3 kinds of typical triangles that we speak of : equilateral, isosceles, and right.

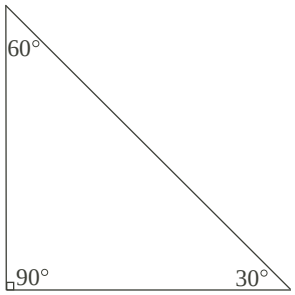
An equilateral triangle is a triangle with all angles that are 60°.




An isosceles triangle is a triangle whose has only two angles which are equal.

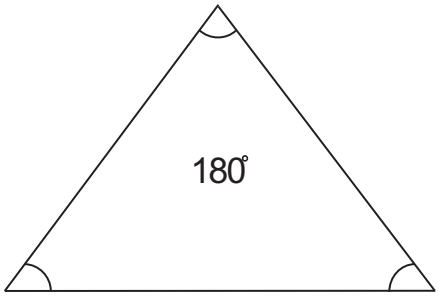


A right triangle is a triangle that has one angle of 90°.

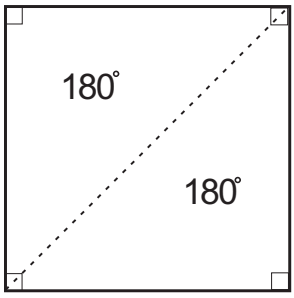



## Sum of Interior Angles

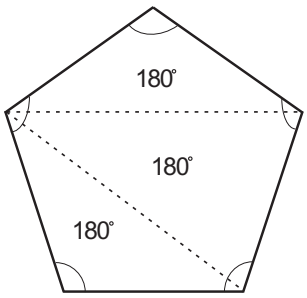
The sum of the interior the angles in a triangle is 180°.




Using this fact, we can then find the sum of interior angles in any polygon. Let's look at a square for example.



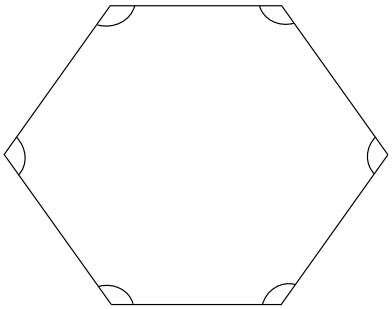

If you start at one corner, and draw a line to other corner of the square, we see the inside of a square is composed of two triangles. So $180° + 180° = 360°$. Meaning the sum of interior angles in a square will always add up to 360°.



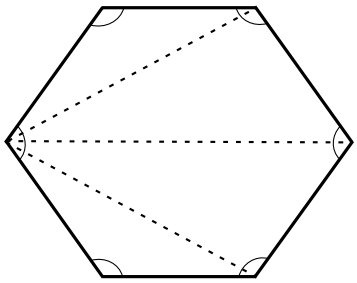

Again, start at one corner of the pentagon, and draw a line to every corner of the shape. The pentagon is made up of 3 triangles. So $3 \times 180° = 540°$.

**Exercise**

Find the sum of interior angles for the hexagon below.



Solution:



Within the hexagon there are 4 triangles. So $4 \times 180° = 720°$.

What pattern do you see?

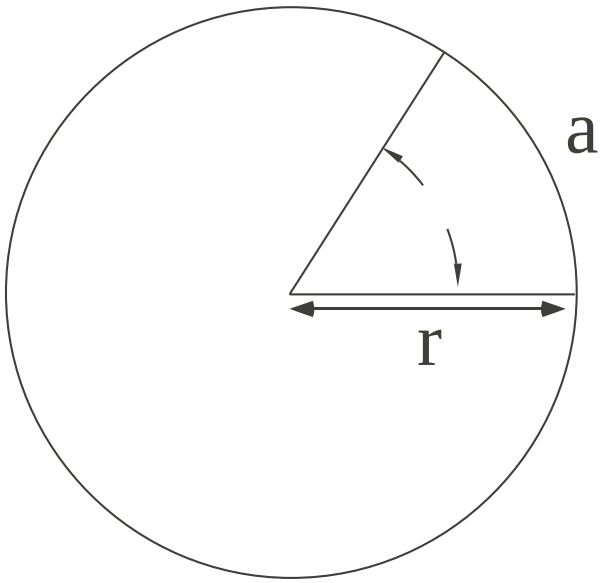
As the shapes grow in size by one side, the number of triangles that can fit into the shape also grows by one triangle. Can you figure out what the formula for the sum of interior angles of a polygon?

$(n-2) \times 180$.

## Radians

A radian is a way of measurement based on the radius of a circle

A radian is given by the angle formed between two radii coming from the center of a circle, to an arc on the circumference of a circle, which has a length of the radius.




i.e. a = r

We denote a radian with $\theta = \frac{a}{r}$

If C = the circumference of a circle then,

$C = \pi\ d = 2\pi r$

So if we want to calculate the radians of an entire circle we have,

$$\theta = \frac{2\pi r}{r} = 2\pi$$

If the radians of an entire circle is $2\pi$, and we know the degrees of circle is 360°, that means,

$$2\pi = 360°.$$

**Exercise**

What do the following radian measure equal to in degrees?

a) $\pi = 180°$

b) $\frac{\pi}{2} = 90°$

c) $\frac{\pi}{4} = 45°$

d) $\frac{3\pi}{2} = 270°$

Convert the degrees into radians.

a) $60° = \frac{\pi}{3}$

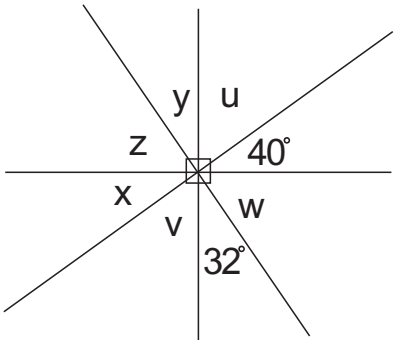
b) $225° = \frac{5\pi}{4}$

c) $135° = \frac{3\pi}{4}$

d) $315° = \frac{7\pi}{4}$

**Problem Set**

1. In the diagram find the degrees of all the angles.



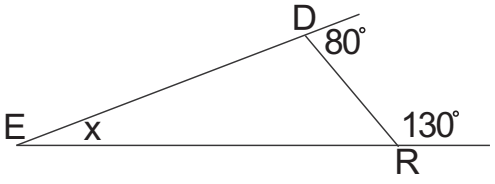

2. What is the angle the hour and minute hand of a clock make when the time is 12:20 p.m.? (Assume the hour hand stays at the hour.)

3. Which of the following set of angles would form an isosceles triangle?

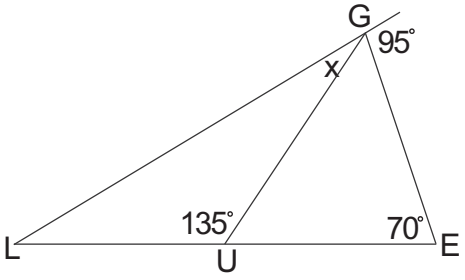
   a) $30°, 60°, 90°$ b) $50°, 30°, 100°$ c) $15°, 150°, 15°$ d) $10°, 75°, 95°$ e) $20°, 20°, 140°$

4. Find the value of x.

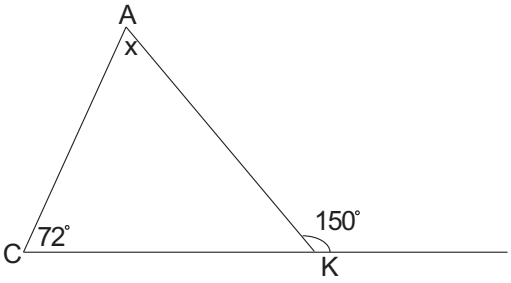
a)



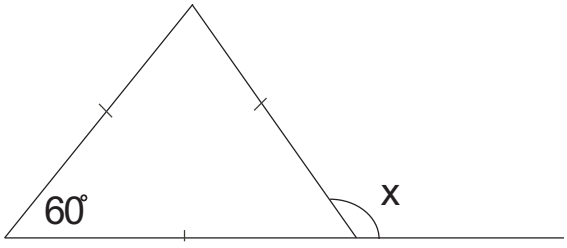

b)



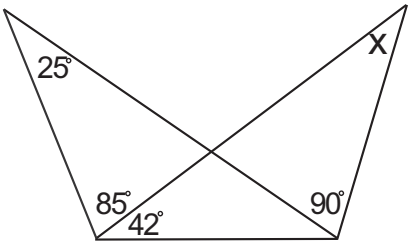

c)



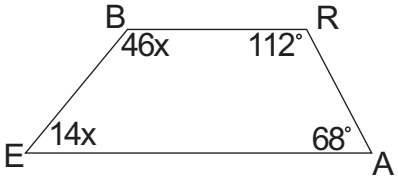

d)



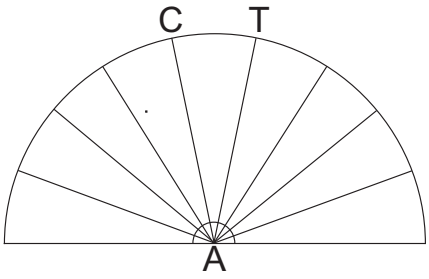

e)



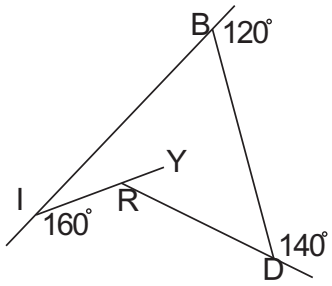

f)



5. A semicircle is split into 9 equal parts. Find the degree of $\angle CAT$.



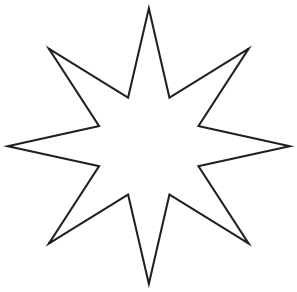

6. In the diagram, what is the degree of $\angle YRD$?




7. Change the degrees into radians, or vice versa.

   a) $15°$
   b) $30°$
   c) $240°$
   d) $300°$
   e) $\frac{5\pi}{2}$
   f) $\frac{8\pi}{2}$
   g) $\frac{9\pi}{4}$
   h) $\frac{2\pi}{3}$

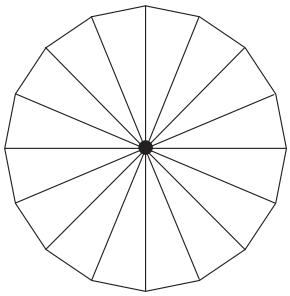
8. Find the sum of interior angles of the star.



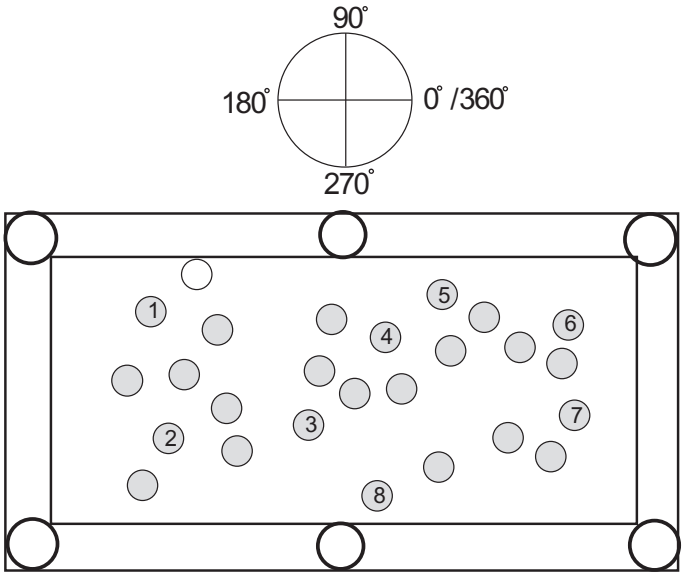
9. What is the sum of interior angles of a decagon?
10. What is the sum of interior angles of a 54 sides figure?
11. What is the sum of interior angles of a circle?

12. As Sammy is driving down the 401, the road becomes very dark and he can't see anything! What should the angle of the light from the lamppost to the ground be so that Sammy can see?

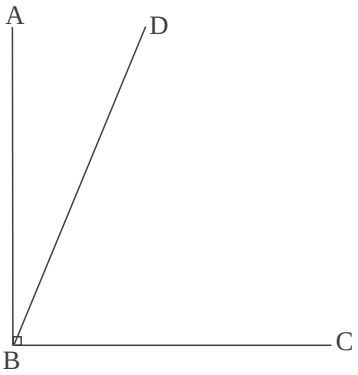
13. There are 16 spokes equally spaced around a wheel. What is the angle between any two spokes in the wheel?



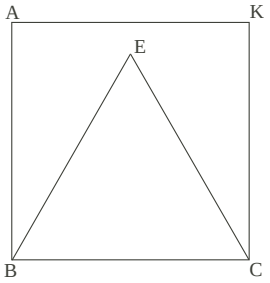
14. Below is another pool table. This time however, the grey balls without numbers are stationary. Meaning they will not move. What are the angles needed to get ball 8 into a pocket? (Hint: You can bounce off objects if you need to.) Approximate answer are fine.



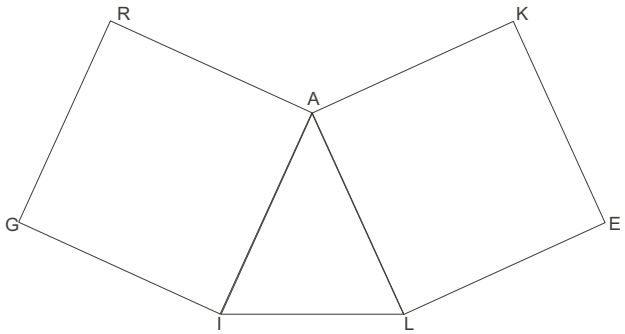

15. In the diagram, $\angle ABC$ is 90°. $\angle CBD$ is 50° larger than $\angle ABD$. What is the degree of $\angle ABD$?



16. $\square BAKC$ is a square and $\triangle BEC$ is an equilateral triangle. What is the degree of $\angle AEK$?




17. $\angle AIL$ is 70° and GRAI and LAKE are equal squares. What is the obtuse angle in $\angle RAK$?

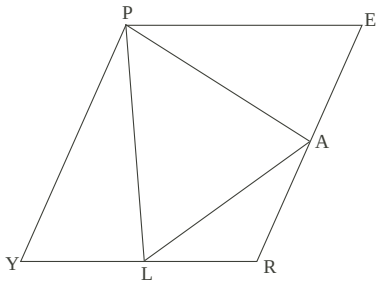



18. In $\triangle DOG$, $\angle O$ is 4 degrees larger than $\angle D$ and $\angle G$ is 2 times $\angle D$. What is the degree of $\angle D$?

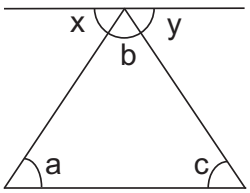
19. Two angles are complementary and one angle is 38° larger than the other one. What is the degree of the smaller angle?

20. The degree of $\angle A$, is twice the degree of $\angle B$, which is twice the degree of $\angle C$. If $\angle A$ and $\angle C$ are supplementary, what is the degree of $\angle B$?

21. PERY is a rhomus. If PY = PL = PA = LA, what is the degree of $\angle YPE$?

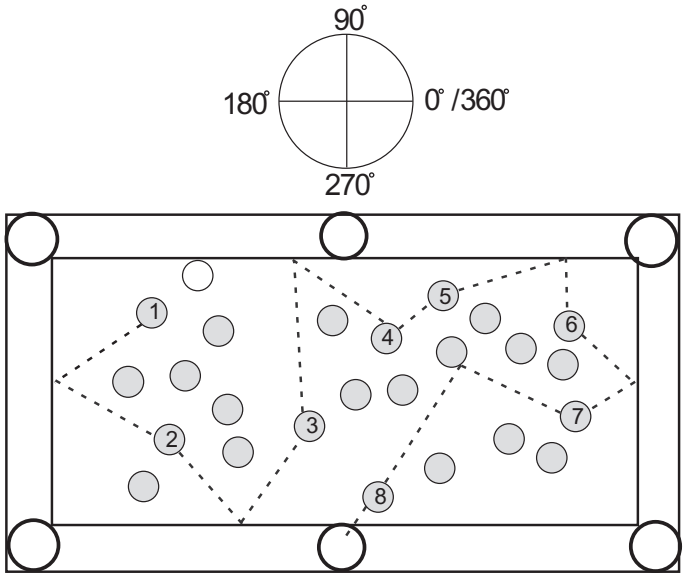



22. Given the diagram below, can you explain why the sum of all the angles in a triangle equal 180°?



**Solutions**

1. x = 40°; y = 32°; u = 50°; v = 50°; w = 58°; z = 58°
2. 110°
3. c) and e)
4. a) x = 30°
   b) x = 20°
   c) x = 78
   d) x = 120°
   e) x = 20°
   f) x = 3
5. $\angle CAT = 20°$
6. 60°
7. a) $\frac{\pi}{12}$; b) $\frac{\pi}{6}$; c) ; d) ; e) 225°; f) 720°; g) 405°; h) 120°
8. (180° × 8) + (6 × 180°) = 2520°
9. (10 - 2) × 180° = 1440°
10. (54 - 2) × 180° = 9360°
11. 360°
12. Between 45° - 70°
13. 22.5°
14. Possible other solutions.



1. Approx. 225°; Approx. 330°
2. Approx. 310°; Approx. 60°
3. Approx. 92°; Approx. 325°
4. Approx. 50°
5. Approx 20°; Approx. 270°
6. Approx 315°; Approx. 210°
7. Approx. 160°; Approx.240°
8. Approx. 240°

15. 20°
16. 150°
17. 140°
18. 46°
19. 26°
20. 72°
21. 100°
22. Extend the base of the triangle, and you see two Z patterns. From there we see x = a, and y = c. Since x + b + c = 180° (supplementary angles), then a + b + c = 180°, as needed.