

Grade 7/8 Math Circles

Winter 2013

3D Geometry

Introductory Problem

Mary's mom bought a box of 60 cookies for Mary to bring to school. Mary decides to bring 30 cookies to school. In how many ways can Mary stack the cookies (on top of one another and/or side by side) to make a box-like figure so it will be easy for her to bring them to school?



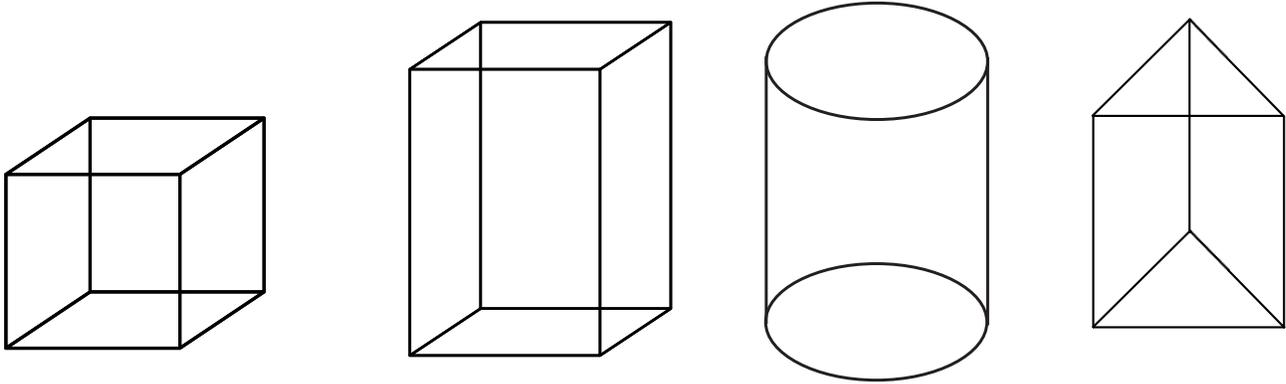
Volume of 3D Figures

The *volume* of a 3D figure is the amount of space within the 3D figure. We measure the volume of any 3D figure in cubic units.

Prisms

Prisms are 3D figures that have congruent parallelogram sides, and a solid base, which is either of two parallel ends on the figure.

Examples



Each figure above is a kind of prism. The first is called a _____. The second is called a _____. The third is a _____, and the fourth is a _____.

The formula to find the volume of a prism is:

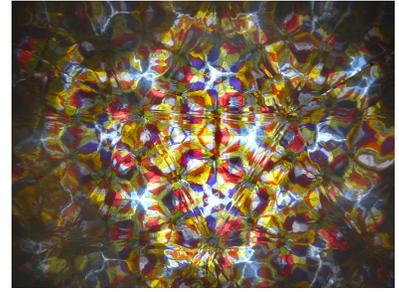
$$V = \text{area of base} \times \text{height}$$

Jack in the Box

Jack's box is 5 cm wide, 5 cm long, and 5 cm tall. How much room does Jack have inside his box?

Kaleidoscope

A kaleidoscope is a cylinder that contains a triangular prism inside made up of mirrors. Inside triangular prism are colorful beads and small pieces of glass. Light reflects off the mirrors of the triangular prism, the beads and the glass so that when a person looks through they see many colors and patterns.



- a) If the dimensions of the triangular prism of a kaleidoscope are a height of 2 cm, a base of 4 cm, and a length of 6 cm, what is the maximum number of beads and glass that can possibly fit in the prism?
- b) What would the height of the cylinder containing the triangular prism have to be? What would be the radius?

c) With all the information you have, what is the volume of the cylinder of this kaleidoscope?

d) How much space is there between the cylinder and the triangular prism?

Cones and Pyramids

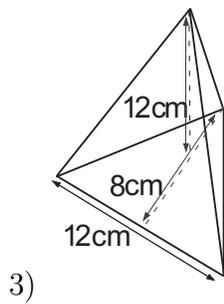
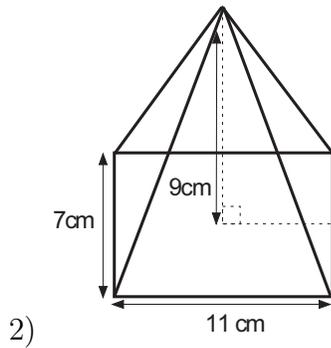
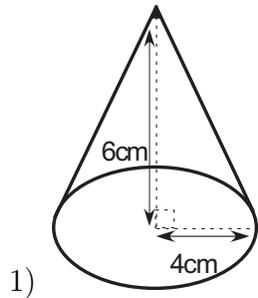
A *cone* is a 3D figure that has a circular base and a rectangular face that wraps around the circumference of the base into a point, called a *common vertex*.

The formula for a cone is: $V = \frac{1}{3} \pi r^2 h$

A *pyramid* is a 3D figure that has a polygon base, and all other triangular faces meet at a common vertex.

The formula for a pyramid is: $V = \frac{1}{3} \times \text{area of base} \times \text{height}$

Exercises



Spheres

A *sphere* is a 3D figure whose surface is at all points equally distant from the center. This distance from the center of the sphere to the surface is called the *radius*.

The formula for the volume of a sphere is: $V = \frac{4}{3}\pi r^3$

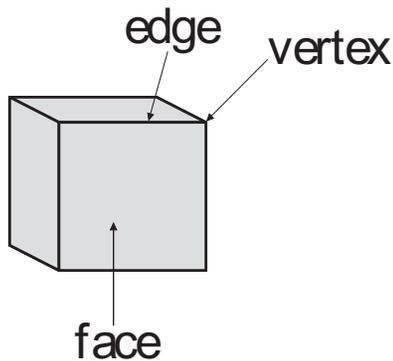
Example

8 soccer balls at Lucy and Eric's school have no air. For being late to class, Lucy and Eric's teacher told them to fill up the 8 soccer balls with air. If the radius of each soccer ball is 12 cm, how much air will they need to fill up the balls?



Euler's Formula

Definitions:



A *polyhedron* is a 3D figure having many faces.

A *face* is any of the shape surfaces on a 3D figure.

An *edge* is a line segment connecting two faces on a polyhedron.

A *vertex* is where three or more edges meet.

Euler was a famous mathematician who discovered a relationship between the faces, edges and vertices of polyhedron. His equation works for all polyhedrons which do not intersect with themselves at some point. We call these kinds of polyhedrons *convex*.

Exercise

Fill out the table below and try to find the relationship between the faces, vertices and edges of polyhedrons.

Names	Faces	Edges	Vertices
Cube			
Rectangular Prism			
Triangular Prism			
Square Based Pyramid			
Pentagonal Prism			
Tetrahedron			
Octahedron			
Dodecahedron			

Euler's Formula is: _____

Surface Area

The *surface area* of a prism is _____. This is true for pyramids and cones as well.

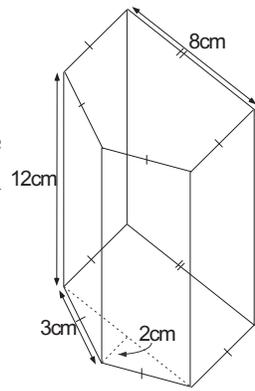
How would you find the surface area of a cube or rectangular prism?

How would you find the surface area of a triangular prism?

How would you find the surface area of a cylinder?

Problem Set

- The Great Pyramids of Giza are named the Pyramid of Menkaure, the Pyramid of Khafre and the Pyramid of Khufu. The height and base of each of these pyramids are: 65.5 m by 103.4 m², 136.4 m by 215.25 m², 138.8 m by 230.4 m² respectively. What is the volume of each of the Great Pyramids of Giza?
- Knowing that an icosahedron has 12 vertices and 30 edges, how many faces does it have?
- Mark wants to build himself a large compost bin for his farm. He measured that he typically has 100m³ of compost a week. Name all the possible combinations of dimensions that he can build his garbage can if the can will be in the shape of a rectangle?
- Forty-two cubes with 2 cm edges are glued together to form a rectangular prism. If the area of the base of the prism is 24 cm and the width of the base is greater than 2 cm, what is the height of the prism?

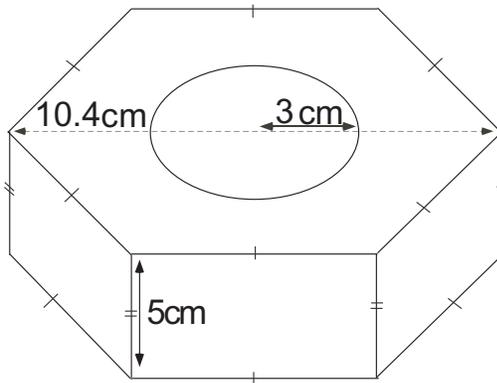


- Daisy bought herself a vase to fill with potpourri for Christmas. If the vase has the dimensions as shown, how much potpourri will she need to fill the vase up to the top?
- Find the smallest cylinder that can fit a cube of 1000 m³.
- A business downtown keeps erasers in boxes with dimensions 24 cm x 28 cm x 13 cm. The erasers have dimensions 2 cm x 4 cm x 1 cm. One of the boxes is half full with erasers.
 - How much room is left in the box to put more erasers?
 - How many erasers can fit into the empty half of the box?
- Looking back at the question about the Great Pyramids of Giza, how much space do the three pyramids take up on the Giza plateau altogether?
- Calculate the amount of metal needed to make 8 cylindrical cans with a diameter of 6 cm and a height of 16 cm.

10. Dean is building a swimming pool in his backyard. The swimming pool will be 18 m long, 24 m wide, and 4.5 m deep. The pool is going to be tiled, with a tile size of 1 m^2 , and it will cost 15 cents per square meter.

- What will it cost to tile the pool?
- How much water can the pool hold?

11. A bolt has a hexagon-like shape head. The bolt bellow has a base perimeter of 36 cm. What is the volume of the head of a bolt with dimensions as shown? What is the surface area?

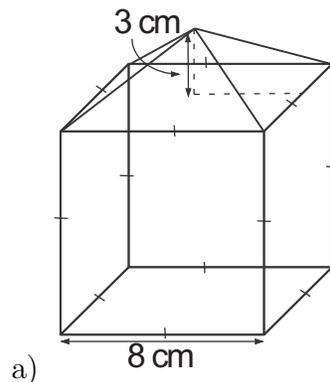


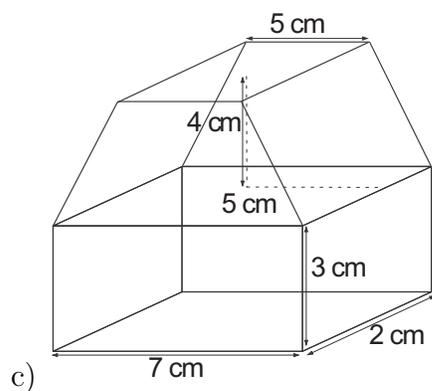
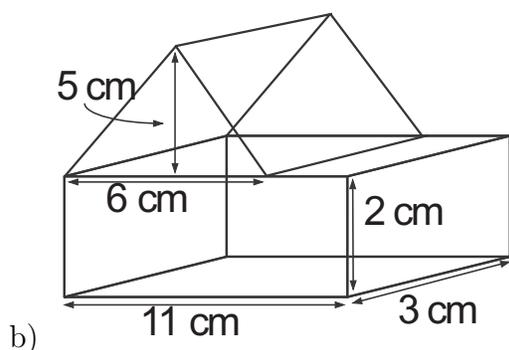
12. A enormous triangular prism holds a triangular shaped chocolate bar. How much cardboard is needed to create a casing that has a base of 12 cm, a height of 8 cm and a length of 20 cm?

13. A hemisphere is half of a sphere. If the radius the hemisphere is 10 mm, what is the volume of the hemisphere?

14. A water pipe section with a volume of 50 m^3 and a diameter of 5 m burst. How long should the sheet metal be to create an identical piece of pipe?

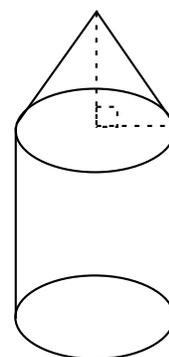
15. Find the volume of the following figures:





16. The volume of a sphere is 70 cm^3 . What is the radius?

17. A small water bottle can hold 389.36 ml of water. Assuming for simplicity, the shape of a typical water bottle is a cylinder with a cone on top, as shown, with a radius of 5 cm . If the total height of the bottle is 6.37 cm , and the height of the cone is half the height of the cylinder, what is the height of the cylinder, and what is the height of the cone?



18. A wedge is a right triangular prism used to prop open doors. Assume a giant wedge is 12 cm long and 8 cm high, with a depth of 8 cm , and is propping a giant door open. If the space between the bottom of a door and the floor is 2 cm , what percentage of the wedge would be under the door?

19. A new tablet is formed through attaching two hemispheres to the ends of a cylinder with a height of 610 mm and radius r . If the volume of the tablet is equal to the volume of a cone of height 189 cm and radius r , find the value of r in mm .