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Grade 7 & 8 Math Circles
Circles Solutions

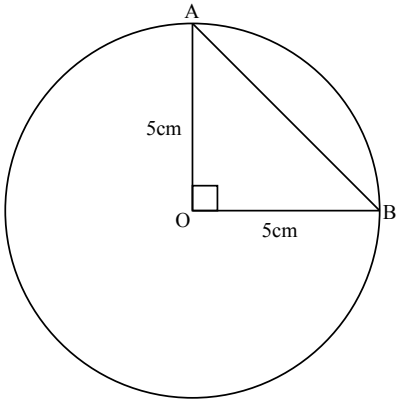
1. Radius = 4cm; Chord = 3cm; Diameter = 8cm; Circumference = 8πcm $\approx$ 25.12cm; Arc Length = $\pi \approx$ 3.14cm; Area of Circle = 16πcm^2 $\approx$ 50.24cm^2; Area of Sector = 2πcm^2 $\approx$ 6.28cm^2; Remember to convert 45° to $\frac{\pi}{4}$ rad when using formulas, or you calculate by knowing we're looking at $\frac{1}{8}$ of a circle.

2. (a) Arc = 6cm; Area = 9cm^2

   (b) Angle = $\frac{3\pi}{4}$ rad; Arc = $\frac{3\pi}{2}$m $\approx$ 4.71m; Area = 3πm^2 $\approx$ 9.42m^2

   (c) Angle = $\frac{\pi}{3}$ rad; Arc = $\frac{25\pi}{3}$cm $\approx$ 26.17cm; Area = $\frac{625\pi}{6}$cm^2 $\approx$ 327.08cm^2

3. Firstly, notice that $\frac{\pi}{2}$rad is a right angle. Using the diagram below, we can find the length of chord AB by using the Pythagorean Theorem. So $AB^2 = AO^2 + BO^2$ which is $AB^2 = 5^2 + 5^2 = 50$. So $AB = \sqrt{50}$cm $\approx$ 7.071cm



4. The large circle has a diameter of 6cm, the diameter of 3 of the smaller circles is also 6cm. Since the smaller circles are all the same size, the diameter of 1 small circle is 2cm. Thus the radius is 1cm.

   So, the area of the larger circle is:

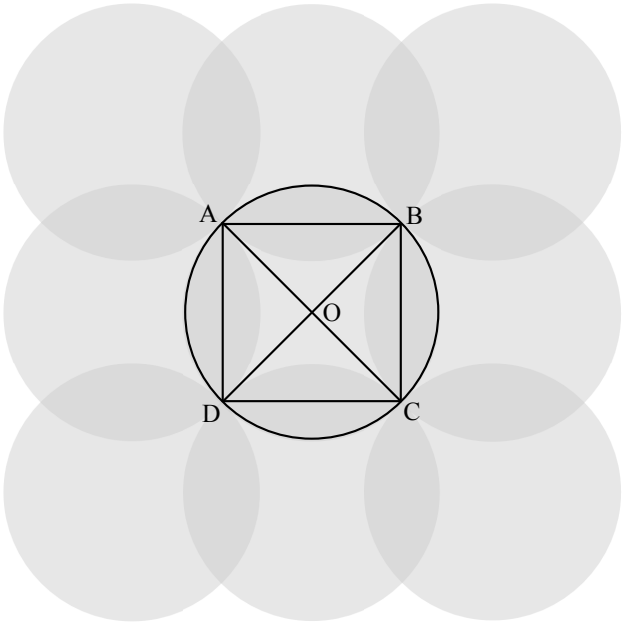
   $A = \pi r^2 = 9\pi \text{cm}^2$

   And the area of 1 smaller circle is:

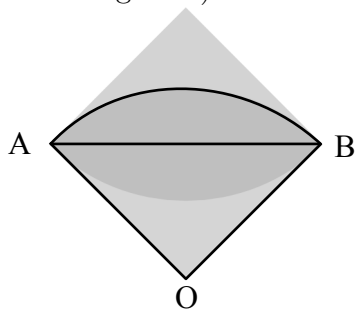
   $A = \pi r^2 = \pi \text{cm}^2$

   Now there are 5 small circles, so the area of the shaded region is $9\pi - 5\pi = 4\pi \text{cm}^2$. Notice that $\pi 2^2 = 4\pi$ So a circle with radius 2 has the same area as the shaded region.

5. Draw two diameter lines which connect opposite petal intersection points. Call these points A, B, C and D. Since all the petals are the same size, these two lines (AC, BD) are perpendicular to one another (important for later). This creates four equal sectors of our circle. We can also create lines AB, BC, CD and DA to form a square. Notice how we have four congruent isosceles triangles with a right angle at the centre of the circle, O.



Let's just look at a quarter of the circle. Notice how the triangle and sector overlap, except for half of the petal (i.e. the segment).



So calculate the area of the segment then multiply by 2, then multiply by the number of petals the get the total shaded area.

$\text{L} = r \times angle = 2 \times \dfrac{\pi}{2} = \pi\text{cm}$

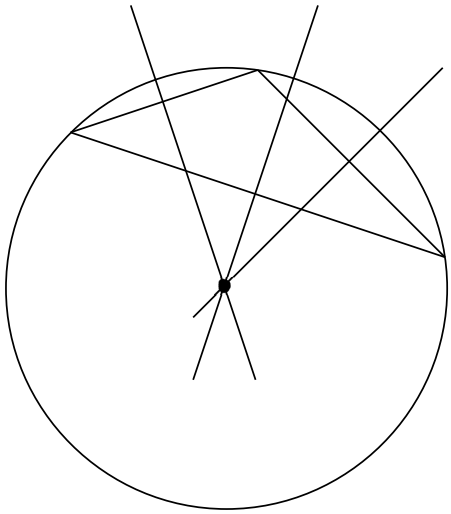
$\text{Sector Area} = \dfrac{r \times L}{2} = \dfrac{2 \times \pi}{2} = \pi\text{cm}^2$

$\text{Triangle Area} = \dfrac{b \times h}{2} = \dfrac{2 \times 2}{2} = 2\text{cm}^2$

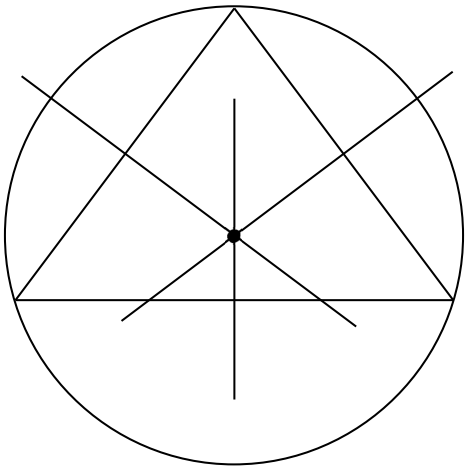
Segment Area = Sector Area - Triangle Area $= (\pi - 2)\text{cm}^2 \approx 1.14\text{cm}^2$

Total Dark Area $\approx 2 \times 1.14 \times 12 = 27.36\text{cm}^2$ (Actual $= 27.39822...\text{cm}^2$)

6. (a) (Note the circles used to create the perpendicular bisectors have been omitted to save space.)



(b)



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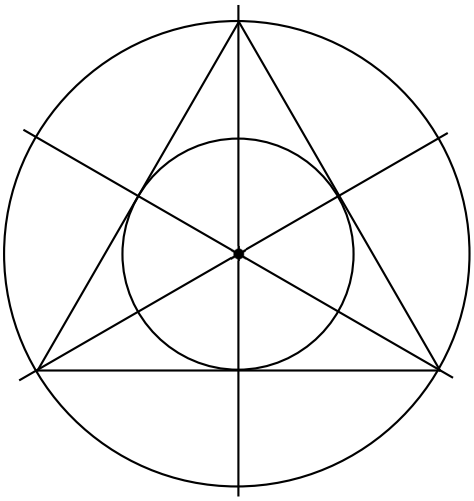

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