UNIVERSITY OF
WATERLOO
Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Enriching Mathematics and
Computer Science for 50 years

Grade 7 & 8 Math Circles
Gauss Contest Preparation

March 26/27, 2013

The Basics

What:

The Gauss Mathematics Contest is an opportunity for students in grades 7 and 8 (and those interested from lower grades), to develop mathematical problem-solving skills and have fun solving a variety of questions. In 2012, over 78 000 students wrote the Gauss contest throughout Canada and worldwide.

There are 2 different contests, one for grade 7 and another for grade 8 students. Participants will write *one* of these contests since both are written at the same time.

There is also a $3.00 fee to write the Gauss. Some schools may pay part or all of the contest fee. Talk to your teacher about the cost when you register.

When:

This year, the contests will be written on **Wednesday, May 15, 2013**. If you are interested in writing the Gauss Contest, you must register at your school by April 18, 2013. The contest is written for 60 minutes at some point throughout the school day.

Where:

The Gauss Contest is written in your school. Just be sure to register by April 18!

Format:

The contest is divided into 3 sections. There are 10 questions in Part A, 10 in Part B, and 5 in Part C. All questions are multiple choice, with 5 different answers each.

A bubble sheet is used to record your responses, as well as all your name, grade, school and other information. It is *very important* that you correctly record all your information, including your answers. Bubbling two answers for one question will be recorded as an incorrect answer. Also, be sure you only use *pencil* on your response form! You are allowed to use rulers, calculators, compasses, and scrap paper on the contest.

The Gauss is designed to have the easiest questions first, then progressively get more challenging. By Part C, questions will require a lot of work and thought. As such, each correct answer is worth: 5 points in Part A, 6 points in Part B, and 8 points in Part C, for a total of 150 points.

Any incorrect answer is worth 0 points. You can earn 2 points for every unanswered question, for up to a total of 10 questions. But why do the markers do that? This prevents you from randomly guessing an answer. If you have no idea how to solve a question, you can automatically get two points for leaving the question blank, as opposed to randomly choosing an answer and hoping it is correct.

The average score on the Gauss is usually around 90 out of 150. You should set a goal as to what mark you'd like to achieve then work towards that goal.

Why write the Gauss?

Because it's fun! Yes I'm serious. The results don't have any affect on your math grade at school, so writing the contest is a personal challenge.

There are a variety of topics covered: geometry, algebra, patterning, data management, measurement, and more. Some questions may also ask you to think in a different way by introducing new ideas which build off content you already know. On the contest, you can

learn cool math tricks (palindromes, magic squares, neat patterns, and more)! You never know what you'll learn when writing a math contest.

The grade 7 and 8 Gauss Contests are the first in a group of contests which can be written through to grade 12. The Gauss is good practice for writing other math contests in the future. You'll know what to expect, and be more prepared for these contests, when there are prizes which can be won.

The Gauss Contests emphasize participation, getting students to enjoy writing math contests and thinking critically. A certificate is provided to every student who writes the contest, and a special recognition certificate is provided to the top scorers in each school. Again though, the Gauss is meant to be a fun and enjoyable experience, not a competition.

How to Register:

Registration must be done through a teacher at your school. Talk to your teacher to see if someone at your school is in charge of registering students for the Gauss Contest. The deadline to register is April 18, 2013.

Some schools may not offer the contest at this time. There are alternatives which can be arranged, but be sure to **speak with your teacher first**!

If you have any questions regarding registration you or your parents can speak to me after class, or your parents can email cemc@math.uwaterloo.ca or call the CEMC office at 519-888-4808.

How to do Well

There are many things you can do in order to do well on the Gauss. Practise solving questions from old contests is the best way to prepare. You can find past contests and solutions on the CEMC website at http://www.cemc.uwaterloo.ca/contests/past_contests.html. Today we will work through some problems from past contests, so you've already started to prepare for the Gauss.

Before you begin writing the test, ensure you have done the following:

- Practised more questions!
- Had a good night's sleep
- Gone to the bathroom (you don't want any interruptions!)
- Had something to eat (so you're not distracted by your stomach)
- Filled a water bottle (be careful not to spill!)
- Sharpened at least 3 pencils and brought plenty of scrap paper
- Tested your calculator
- Brought a **good** eraser with you

By doing all of the above you will ensure you have as few distractions as possible so you can concentrate on solving problems.

Writing the Contest

Here's a strategy I like to go through when working on the contest. It makes the most effective use of your time; remember you only have 60 minutes.

All questions in Part A should be fairly easy, although the difficulty will increase as the question numbers increase. Quickly try all the questions in Part A first. If you get stuck on one question or are spending more than about a minute on it (unless you're close to an answer), skip over it for now. Once you've completed question 10, return to any skipped questions, and spend a bit more time on them. If you're still spending too much time on one question or are stuck, leave that answer blank for now.

Since Part A is the simplest section of the contest, you should be able to get through it fairly quickly. But be sure that you're confident with your answers; these should be easy marks. It's better to leave a question blank and get two points or spend a bit of time making sure your answer is correct, rather than rush through and make a silly mistake.

Part B starts to get a bit harder. You will need to do write down more rough work than in Part A and use multiple steps. Go through the same strategy as Part A, but take a bit more time if needed. Again, if you get stuck, don't know where to begin or are spending more than a few minutes working on a problem, skip it for now. Return to those questions after you've looked at all questions from Part B. Leave any responses blank if you are still stuck after you've looked at the questions again.

By this point, either you've answered each question and are confident with your responses, or you've looked at any blank question at least twice in Parts A and B. Quickly reread these skipped problems and try to work through them if possible. Leave them blank for the final time if you still cannot solve these problems.

Now move on to Part C. These questions will require you to write quite a bit down as rough work, and also require the most time and effort. This is why you need to be careful about how much time you spend on the easier questions. The faster you work through Parts A and B, while still being confident with your answers, the more time you will have to work on the harder Part C questions.

Read over all the questions in Part C (questions 21 to 25). If you have an idea of how to do one or more of the questions, go right to that one and work through the problem. Again these questions are more complex and will require many steps. It will take a longer time to solve these questions. Use all your remaining time to work on questions from Part C.

You may not finish all the problems before the 60 minutes are up. This is okay! Since you've worked through all the easiest problems you will get points for most or all (as long as you've done the math correctly) of those answers. Questions you didn't answer will give you 2 points each, for up to 10 questions. If you follow this process, you will maximize your score and work through as many questions as possible.

Once time runs out, congratulate yourself; you'll have completed the Gauss Contest!

Problem Solving

All this information is helpful, but you still need to actually solve each problem. Now we will look at a few problems and some strategies you could use to solve them. All the problems below come from past Gauss contests. The code before each question (e.g. 2010-G7-19) refers to the year, contest (Grade 7 Gauss or Grade 8 Gauss), and question number.

Hint # 1: Answer the question without reading any of the answers. This strategy works really well with many questions in Part A and some in Part B.

2012-G7-3 & 2012-G8-2: A six-sided die has the numbers one to six on its sides. What is the probability of rolling a five?
I think the answer is _____.

Only after you've written down your answer, check to see if your answer matches up with any of the choices. Note the possible answers are upside down to prevent any wandering eyes from peeking before you have your answer. On the contest, all answers are right-side up.

(A) 2/6 (B) 1/6 (C) 5/6 (D) 3/6 (E) 4/6

Hopefully the answer you calculated is also one choice. If so, you are likely correct and so you should choose that response. If your answer is not one of the choices you know your answer is incorrect, so you can retry the problem.

Again on this next example, calculate an answer first then check to see if it's also a choice.

2012-G7-17: The ratio of boys to girls at Gauss Public School is 8 : 5. If there are 128 boys at the school, then how many students are there at the school? _____

After you've calculated an answer, check the choices below.

(A) 218 (B) 253 (C) 208 (D) 133 (E) 198

Hint # 2: Reread the question.

Many questions will have a lot of numbers and words. Make sure you understand what is being asked. You may need to reread the question many times. If the question requires

multiple steps, you may want to reread the question after completing a step.

2012-G8-5: How many more coins does it take to make one dollar (100¢) using only nickels (5¢ coins) than it takes to make one dollar using only dimes (10¢ coins)?

(A) 15 (B) 10 (C) 25 (D) 5 (E) 20

Hint # 3: Eliminate some of the choices.

You *will* get stuck on some problems. One way to help yourself is to get rid of some answers you know are wrong. You can do this in a few ways.

Many times the writers of the contest will try to trick you by creating some of the incorrect choices using common errors. If you see a question with exponents, the the writers will make one of the wrong choices as if the the student multiplied the base and exponent together. So if a question asks, "What is 3^2?" one choice would be 6 (since $3 \times 2 = 6$). If you go through the question and intentionally making a common mistake, you can eliminate that answer.

For the following question, eliminate the wrong answers by explaining the common mistakes students might make to get each answer.

2012-G7-11: The perimeter of a square is 36 cm. The area of the square, in cm^2, is

(A) 24

(B) 81

(C) 36

(D) 1296

(E) 324

If you're working through a problem, you may also find at certain stages you can eliminate some choices. Here's another example:

2011-G7-22: A pool has a volume of 4000 L. Sheila starts filling the empty pool with water at a rate of 20 L/min. The pool springs a leak after 20 minutes and water leaks out at 2 L/min. Beginning from the time when Sheila starts filling the empty pool, how long does it take until the pool is completely full?

(A) 3 hrs (B) 3 hrs 40 mins (C) 4 hrs (D) 4 hrs 20 mins (E) 3 hrs 20 mins

This is a Part C question, so it will challenge a lot of people. But we can eliminate a few answers to get you started. How long would it take to fill the pool if the leak *doesn't* occur?

But the leak does occur, so how will this affect the actual time it takes to fill the pool? So what answers can we eliminate?

Hint # 4: Test the choices to see if they satisfy the conditions.

This method works especially well once you've already eliminated some possibilities. You simply take one of the choices (usually you'd start with A) and see if it works with the numbers given in the question. In a way you are doing the question in reverse. Note that for many questions it's not possible to use this hint.

Perform Clara's operations on each of the possible choices below. Make sure to test them all, since you might make a mistake and stop too soon. If you happen to get two (or more)

answers which seem to work out correctly, you'll know you've made an error.

2011-G8-6: If Clara doubles a number and then adds 3, the result is 23. The original number is

(A) 13

(B) 10

(C) 49

(D) 17

(E) 20

Hint # 5: Draw a diagram.

Certain types of math, such as geometry, are very visual. It's more difficult work with geometry when words are used to describe a scenario. By drawing a diagram based on the given description, you can better understand the problem.

A quick note about diagrams: Many questions will have diagrams included. These diagrams are **NOT DRAWN TO SCALE!** This means if in the description of the problem, some angle is given as 45° and a line is said to be 23cm long, in the diagram the angle won't be exactly 45° and the line will be shorter than 23cm. You may use the diagram given or one that you draw to help understand the math behind the problem, but you cannot simply measure the needed quantity directly from your diagram!

Draw a diagram to help you solve the following problem.

2012-G8-6: Ronald buys a pizza cut into 12 equal parts. He then cuts each part into 2 equal pieces. If he eats 3 of these pieces, what fraction of the pizza does he eat?

(A) $\frac{1}{24}$ (B) $\frac{1}{2}$ (C) $\frac{3}{8}$ (D) $\frac{1}{6}$ (E) $\frac{1}{8}$

Remember that your diagram doesn't need to be exact! Even if it's not exact, it can still help you solve the problem and it shouldn't take too much time to draw.

Hint # 6: Look for patterns.

This tip is great for the later questions. If you need to write out all the possibilities, or draw a complex diagram to solve a problem, usually there is some pattern which will make the question easier to solve. It's not always easy to find a pattern. There are so many types of patterns and ways patterns can be hidden in a problem. Finding a pattern will require creativity and a bit of trial and error. Let's take a difficult example, that becomes easy once we see a pattern.

2011-G7-25: Ten circles are all the same size. Each pair of these circles overlap but no circle is exactly on top of another circle. What is the greatest possible total number of intersection points of these ten circles?

(A) 40 (B) 70 (C) 80 (D) 90 (E) 110

Hint # 7: Write out what you are given.

This is a great way to organize information. It turns word problems into mathematical questions. It also allows you to organize information any way you like: you can put all similar numbers together, draw mini diagrams, and so on. This organization might lead you think of certain ways to work out a problem.

Write out and organize the information given to you in the question below, turning it from a word problem into a math based question, then find the answer.

2010-G8-18: A bicycle travels at a constant speed of 15 km/h. A bus starts 195 km behind the bicycle and catches up to the bicycle in 3 hours. What is the average speed of the bus in km/h?

(A) 65 (B) 80 (C) 70 (D) 60 (E) 50

Hint # 8: Ask yourself "What am I given & what can I do with it?" and "Why are they asking this question?"

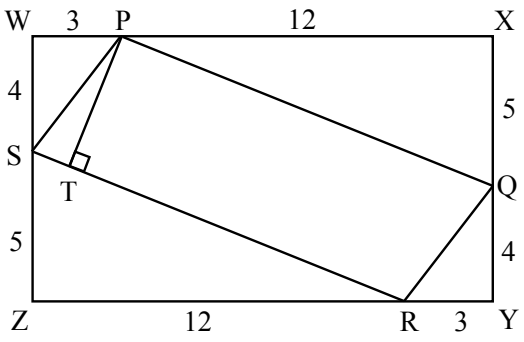
Hint # 7 and Hint # 8 are very closely related. In fact, follow Hint 7 before using Hint 8.

Many times, there will be a hint given in a question or a diagram. However this hint will be hidden. By asking these questions you will find the hint, and start working towards a solution.

For the question below, we don't want to find an answer (yet). Answer the above questions first then think of ways you could approach the solution.

2012-G8-25: In the rectangle WXYZ, the parallelogram PQRS is formed as shown. The segment PT is perpendicular to SR. The length of ST is

(A) $\frac{13}{12}$ (B) $\frac{13}{5}$ (C) $\frac{12}{13}$
(D) $\frac{16}{13}$ (E) 1



"What information are you given?" Think about it specifically (eg. length PX = 12) but only write down the general idea if there are a lot of similar information (eg. lengths of lines).

"Why are the writers asking this question?" You could also ask yourself, "What mathematical concepts might the writers want to test?"

"What can you do with this information?"

Hint # 9: Once you have an answer, reread the question. Make sure you've answered what the question is asking.

This may sound like a silly hint but most questions will have multiple parts. You may calculate a number and think it is the final answer, when you actually forgot to use some of the given information. When rereading the question, make sure you've used all the given information and that your final answer responds to the actual question.

Good Luck!

Now you have the tools required to write the Gauss Contest. You will likely come up with your own strategies for solving problems as you practise for and write the Gauss, and future math contests. Good luck and remember to have fun!

Additional Problems

Note that all problems are from past Gauss Contests. The code before each question (e.g. 2010-G7-19) refers to the year, contest (Grade 7 Gauss or Grade 8 Gauss), and question number. You can view more problems from past contests and *full solutions* on the CEMC website at http://www.cemc.uwaterloo.ca/contests/past_contests.html

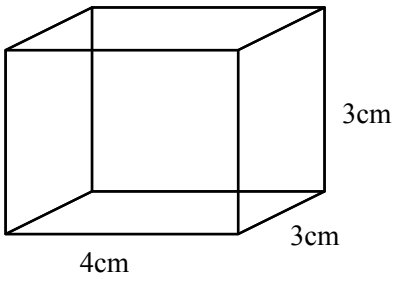
2010-G7-3: The value of $4 \times 5 + 5 \times 4$ is

(A) 160 (B) 400 (C) 100 (D) 18 (E) 40

2010-G7-6: If $10.0003 \times ? = 10000.3$, the number that should replace the $?$ is

(A) 100 (B) 1000 (C) 10000 (D) 0.001 (E) 0.0001

2010-G7-8: How many 1 cm $\times$ 1 cm $\times$ 1 cm blocks are needed to build the solid rectangular prism shown?



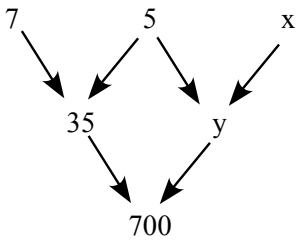
(A) 10 (B) 12 (C) 33

(D) 66 (E) 36

2010-G8-9: Chris was given $\frac{1}{3}$ of the 84 cookies in the cookie jar. He ate $\frac{3}{4}$ of the cookies that he was given. How many cookies did Chris eat?

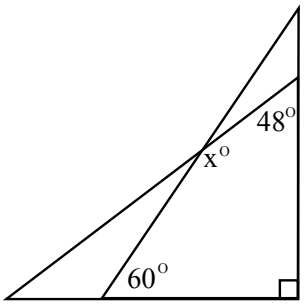
(A) 36 (B) 48 (C) 35 (D) 28 (E) 21

2010-G7-10: Each number below the top row is the product of the number to the right and the number to the left in the row immediately above it. What is the value of x?



(A) 8 (B) 4 (C) 7 (D) 5 (E) 6

2010-G8-10: In the diagram, the value of x is



(A) 72 (B) 158 (C) 108 (D) 138 (E) 162

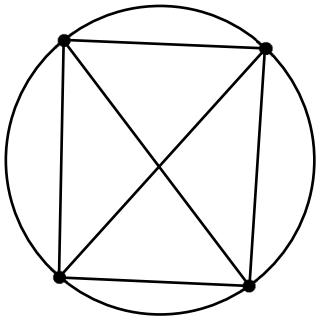
2011-G7-14: A palindrome is a positive integer that is the same when read forwards or backwards. For example, 545 and 1331 are both palindromes. The difference between the smallest three-digit palindrome and the largest three-digit palindrome is

(A) 909 (B) 898 (C) 888 (D) 979 (E) 878

2010-G7-17: How many three-digit integers are exactly 17 more than a two-digit integer?

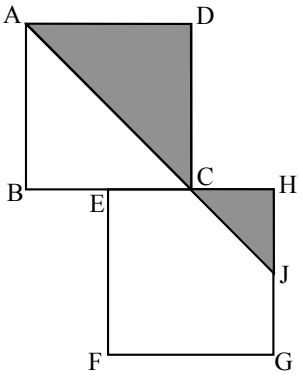
(A) 17 (B) 16 (C) 10 (D) 18 (E) 5

2010-G8-17: Distinct points are placed on a circle. Each pair of points is joined with a line segment. An example with 4 points and 6 line segments is shown. If 8 distinct points are placed on a circle, how many line segments would there be?



(A) 27 (B) 36 (C) 21 (D) 56 (E) 28

2011-G8-18: Squares ABCD and EFGH are equal in area. Vertices B, E, C, and H lie on the same straight line. Diagonal AC is extended to J, the midpoint of GH. The fraction of the two squares that is shaded is



(A) $\frac{5}{8}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{5}{16}$ (E) $\frac{3}{8}$

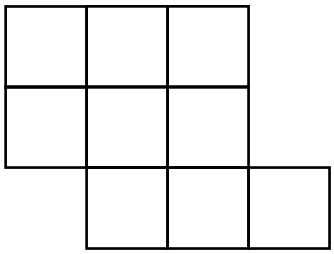
2009-G7-23: Each time Kim pours water from a jug into a glass, exactly 10% of the water remaining in the jug is used. What is the minimum number of times that she must pour water into a glass so that less than half the water remains in the jug?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

2010-G8-23: To shovel all of the snow on his driveway, Kevin needs 12 hours. Individually, Dave needs 8 hours to shovel all of Kevin's snow, John needs 6 hours to shovel all of Kevin's snow, and Allison needs 4 hours to shovel all of Kevin's snow. If Kevin, Dave, John, and Allison all work together, how many minutes do they need to shovel all of Kevin's snow?

(A) 108 (B) 120 (C) 84 (D) 96 (E) 90

2011-G7-24: From the figure shown, three of the nine squares are to be selected. Each of the three selected squares must share a side with at least one of the other two selected squares. In how many ways can this be done?



(A) 19 (B) 22 (C) 15 (D) 16 (E) 20

2010-G8-24: Two circles each have radius 10 cm. They overlap so that each contains exactly 25% of the other's circumference. The area of the region which they overlap is closest to

(A) 57.08 cm^2 (B) 55.24 cm^2 (C) 51.83 cm^2 (D) 54.17 cm^2 (E) 53.21 cm^2