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Enriching Mathematics and Computer Science for 50 years

Grade 6 Math Circles
Algebra - Solutions

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Exercise 1

Using BEDMAS, we get the following:

A. $$\begin{aligned} 11 - 4 + 13 \times 2 &= 11 - 4 + 26 \\ &= 7 + 26 \\ &= 33 \end{aligned}$$

B. $$\begin{aligned} 5 - 45 \div 15 + \frac{3}{2} &= 5 - 3 + \frac{3}{2} \\ &= 2 + \frac{3}{2} \\ &= \frac{4}{2} + \frac{3}{2} \\ &= \frac{4+3}{2} \\ &= \frac{7}{2} \end{aligned}$$

C. $$\begin{aligned} 4 \times 5 \div 2 + 7 - 4 \times 4 &= 20 \div 2 + 7 - 4 \times 4 \\ &= 10 + 7 - 4 \times 4 \\ &= 10 + 7 - 16 \\ &= 17 - 16 \\ &= 1 \end{aligned}$$

D. $$\begin{aligned} 42 \div 7 \times 3 + 9 \times 2 - 4 &= 6 \times 3 + 9 \times 2 - 4 \\ &= 18 + 9 \times 2 - 4 \\ &= 18 + 18 - 4 \\ &= 36 - 4 \\ &= 32 \end{aligned}$$

Exercise 2

Using BEDMAS, we get the following:

A. $$\begin{aligned} 17 - 2^3 + 4 \times 5 &= 17 - 8 + 4 \times 5 \\ &= 17 - 8 + 20 \\ &= 9 + 20 \\ &= 29 \end{aligned}$$

B. $$\begin{aligned} 72 \div (3 \times 2) - (10 + 1) &= 72 \div 6 - 11 \\ &= 12 - 11 \\ &= 1 \end{aligned}$$

C. $$\begin{aligned} 10 \times (0.4 + 0.3) - 2^2 \div 5^0 &= 10 \times 0.7 - 2^2 \div 5^0 \\ &= 10 \times 0.7 - 4 \div 1 \\ &= 7 - 4 \\ &= 3 \end{aligned}$$

D. $$\begin{aligned} 10 \times 0.4 + 0.3 - 2^2 \div 5^0 &= 10 \times 0.4 + 0.3 - 4 \div 1 \\ &= 4 + 0.3 - 4 \\ &= 0.3 \end{aligned}$$

E. $$\begin{aligned} 6 + (36 \div 9)^3 \div 2 - 1 &= 6 + (4)^3 \div 2 - 1 \\ &= 6 + 64 \div 2 - 1 \\ &= 6 + 32 - 1 \\ &= 37 \end{aligned}$$

F. $$\begin{aligned} (14 - 6) \times ((30 + 5) \div 5) &= (14 - 6) \times (35 \div 5) \\ &= 8 \times 7 \\ &= 56 \end{aligned}$$

G. $$\begin{aligned} (95 \div 19)^2 + 3 &= (5)^2 + 3 \\ &= 25 + 3 \\ &= 28 \end{aligned}$$

H. $$\begin{aligned} 3 \times 7 + 5 - 100 \div 20 \times 4 &= 21 + 5 - 5 \times 4 \\ &= 21 + 5 - 20 \\ &= 6 \end{aligned}$$

Exercise 3

Using our 2-step method to solving algebraic equations, we get the following:

A. $$\begin{aligned} x - 4 &= 5 \\ x - 4 \ {\color{red}+4} &= 5 \ {\color{red}+4} \\ x &= 9 \end{aligned}$$

B. $$\begin{aligned} 2x &= 22 \\ \frac{2x}{{\color{red}2}} &= \frac{22}{{\color{red}2}} \\ x &= 11 \end{aligned}$$

C. $$\begin{aligned} \frac{x}{3} &= 7 \\ \left(\frac{x}{3}\right) {\color{red}(3)} &= (7){\color{red}(3)} \\ x &= 21 \end{aligned}$$

D. $$\begin{aligned} 6 &= x + 1 \\ x + 1 &= 6 \\ x + 1 \ {\color{red}-1} &= 6 \ {\color{red}-1} \\ x &= 5 \end{aligned}$$

E. $$\begin{aligned} 2x - 3 &= 5 \\ 2x - 3 \ {\color{red}+3} &= 5 \ {\color{red}+3} \\ 2x &= 8 \\ \frac{2x}{{\color{red}2}} &= \frac{8}{{\color{red}2}} \\ x &= 4 \end{aligned}$$

F.
$$\begin{aligned} 3 &= 1 + 2x \\ 1 + 2x &= 3 \\ 1 \ {-1} + 2x &= 3 \ {-1} \\ 2x &= 2 \\ \frac{2x}{2} &= \frac{2}{2} \\ x &= 1 \end{aligned}$$

G.
$$\begin{aligned} 3x &= 2x + 4 \\ 3x \ {-2x} &= 2x \ {-2x} + 4 \\ x &= 4 \end{aligned}$$

H.
$$\begin{aligned} 5x + 1 &= 2x + 28 \\ 5x + 1 \ {-1} &= 2x + 28 \ {-1} \\ 5x &= 2x + 27 \\ 5x \ {-2x} &= 2x \ {-2x} + 27 \\ 3x &= 27 \\ \frac{3x}{3} &= \frac{27}{3} \\ x &= 9 \end{aligned}$$

Problem Set

1. Using BEDMAS, we get the following:

$$\begin{aligned} (4)(2+1) - 7 + ((8)(9))^0 - (2)\left(\frac{97+5}{51}\right) &= (4)(3) - 7 + 72^0 - (2)\left(\frac{102}{51}\right) \\ &= (4)(3) - 7 + 72^0 - (2)(2) \\ &= (4)(3) - 7 + 1 - (2)(2) \\ &= 12 - 7 + 1 - 4 \\ &= 2 \end{aligned}$$

2. The sum of 6 and 8 is $6 + 8 = 14$.
   The product of 6 and 2 is $(6)(2) = 12$.
   The difference is $14 - 12 = 2$.
   Therefore, the sum of 6 and 8 is greater than the product of 6 and 2 by 2.

3. 5 nickels is $(5)(5) = 25$ cents.
   5 dimes is $(5)(10) = 50$ cents.
   5 quarters is $(5)(25) = 125$ cents.
   Therefore, Victoria has a total of $25 + 50 + 125 = 200$ cents, or \$2.00, in her pocket.

4. (a) If we think of each album as a group of 8 songs, then Jimmy bought 8 groups of 8 songs. Therefore, Jimmy bought $(8)(8) = 8^2 = 64$ songs.

   (b) Listening to all of the albums in sequence means listening to 64 songs. Since each song is approximately 4 minutes, it will take Jimmy $(4)(64) = 256$ minutes to listen to all the albums in sequence.

   (c) Jimmy has $7 - 4 = 3$ hours to listen to his music before dinner. We could also say that he has $(3)(60) = 180$ minutes to listen to his music before dinner since there are 60 minutes in an hour. Since $180 < 256$, Jimmy will not be able to listen to all of his albums before dinner.

5. (a) Answers may vary.

   (b) Notice that the amount of money received each day follows a pattern: *Each day's payment is twice the amount of previous day's payment.* Or in other words, we are multiplying the previous day's payment by 2 to calculate the current day's payment.
   Consider writing the payments in the form

$$(\text{day } 1) = \$2^0,\ (\text{day } 2) = \$2^1,\ (\text{day } 3) = \$2^2,\ (\text{day } 4) = \$2^3, \ldots$$

   That is, on the nth day, you receive 2^{n-1} dollars in winnings.
   Therefore, on the 25th day, you receive $2^{25-1} = 2^{24} = 16,777,216$ dollars in winnings.

   (c) We already know that Option A yields 30 million dollars. So we just need to figure out whether Option B yields more, less, or the same amount of money.
   Note that with Option B, you *receive* 2^{n-1} dollars on the nth day. The total winnings from Option B is the *sum* of all of those daily payments!
   Rather than computing the sum beginning at the first day, lets start adding up terms from the 25th day and go backwards. That way, we will quickly get a better

idea of whether or not the yield of Option B will approach 30 million.

$$\begin{aligned} 2^{24} &= 16,777,216 \\ 2^{24} + 2^{23} &= 25,165,824 \\ 2^{24} + 2^{23} + 2^{22} &= 29,360,128 \\ 2^{24} + 2^{23} + 2^{22} + 2^{21} &= 31,457,280 \end{aligned}$$

So just in the last 4 of the 25 days, Option B yields more than 30 million dollars. Thus, we can see that Option B will yield more money than Option A.

6. Think of the equation like a balance. By symmetry, we must have that $x = 18$.

7. Simplifying the equation first, we get the following:

$$\begin{aligned} \frac{(3)(9) - 10}{2} &= \frac{34}{x} \\ \frac{27 - 10}{2} &= \frac{34}{x} \\ \frac{17}{2} &= \frac{34}{x} \end{aligned}$$

Now notice that the fractions are equivalent if $x = 4$.

8. Using our 2-step method to solving algebraic equations, we get the following:

(a)
$$\begin{aligned} 5x + (7)(3) &= 39 - x \\ 5x + 21 &= 39 - x \\ 5x + 21 \ {\color{red}-21} &= 39 \ {\color{red}-21} - x \\ 5x &= 18 - x \\ 5x \ {\color{red}+x} &= 18 - x \ {\color{red}+x} \\ 6x &= 18 \\ \frac{6x}{{\color{red}6}} &= \frac{18}{{\color{red}6}} \\ x &= 3 \end{aligned}$$

(b)
$$\begin{aligned}
\frac{x}{(1+2^2)} &= (13-12+11-10)\left(\frac{30}{6}\right)\\
\frac{x}{(1+4)} &= (2)(5)\\
\frac{x}{5} &= (2)(5)\\
\frac{x}{5} &= 10\\
\left(\frac{x}{5}\right)(5) &= (10)(5)\\
x &= 50
\end{aligned}$$

(c)
$$\begin{aligned}
\left(\frac{52-34}{6}\right)^3 x + 8 &= 40x - (7+24)\\
\left(\frac{18}{6}\right)^3 x + 8 &= 40x - 31\\
3^3 x + 8 &= 40x - 31\\
27x + 8 &= 40x - 31\\
27x - 27x + 8 &= 40x - 27x - 31\\
8 &= 13x - 31\\
8 + 31 &= 13x - 31 + 31\\
39 &= 13x\\
\frac{39}{13} &= \frac{13x}{13}\\
3 &= x\\
x &= 3
\end{aligned}$$

9. Let x be the unknown number. Then we must solve the following equation:

$$7x = x + 36$$

Using our 2-step method to solving algebraic equations, we get the following:

$$\begin{aligned} 7x &= x + 36 \\ 7x \ {\color{red}-x} &= x \ {\color{red}-x} + 36 \\ 6x &= 36 \\ \frac{6x}{{\color{red}6}} &= \frac{36}{{\color{red}6}} \\ x &= 6 \end{aligned}$$

10. Let x be the number of freight cars in the train. Then, we have the equation

$$21 + 11x = 362$$

Using our 2-step method to solving algebraic equations, we get the following:

$$\begin{aligned} 21 + 11x &= 362 \\ 21 \ {\color{red}-21} + 11x &= 362 \ {\color{red}-21} \\ 11x &= 341 \\ \frac{11x}{{\color{red}11}} &= \frac{341}{{\color{red}11}} \\ x &= 31 \end{aligned}$$

That is, there are 31 *freight cars* in the train. But the question asked for the total number of cars in the train, including the engine. So the correct answer is that there are $31 + 1 = 32$ cars in the train.

11. One-third of 48 is 16 since $16 + 16 + 16 = 48$. One-quarter of 48 is 12 since $12 + 12 + 12 + 12 = 48$. Therefore, three-quarters of 48 is $12 + 12 + 12 = 36$.
Let x be the amount of gas (in litres) that Kathy must add. Then we can create an equation to describe the question:

$$16 + x = 36$$

We can easily solve this equation:

$$\begin{aligned} 16 \ -16 + x &= 36 \ -16 \\ x &= 20 \end{aligned}$$

So Kathy must add 20 L of gas to the tank.

12. Let x be the largest of the three consecutive integers.
    Then the next smallest integer is $x - 1$. And the smallest of the three consecutive integers is $x - 2$.
    Since the sum of the three integers is 90, we can write the following equation:

$$x + x - 1 + x - 2 = 90$$

Using our 2-step method to solving algebraic equations, we get the following:

$$\begin{aligned} x + x - 1 + x - 2 &= 90 \\ 3x - 3 &= 90 \\ 3x - 3 \ +3 &= 90 \ +3 \\ 3x &= 93 \\ \frac{3x}{3} &= \frac{93}{3} \\ x &= 31 \end{aligned}$$

So the largest of the three consecutive integers described is 31.

13. Let x be the number of sour keys that Nathan bought.
    Since Nathan bought twice as many gumdrops than sour keys, he bought $2x$ gumdrops.
    We can write the equation below to describe the question (use cents):

$$(10)(2)x + 30x = 300$$

Using our 2-step method to solving algebraic equations, we get the following:

$$\begin{aligned}
(10)(2)x + 30x &= 300 \\
20x + 30x &= 300 \\
50x &= 300 \\
\frac{50x}{50} &= \frac{300}{50} \\
x &= 6
\end{aligned}$$

Therefore, Nathan bought 6 sour keys from the store.

14. Let the longer sides of the rectangle $(x + 4)$ be referred to as the *length*, and let the shorter sides $(x - 2)$ be referred to as the *width*.
The area of a rectangle is $length \times width$. But we need to know x in order to compute this.
Thankfully, we can find x from the perimeter!
The perimeter of a rectangle is given by $2 \times length + 2 \times width$. Therefore, we have that

$$x + 4 + x + 4 + x - 2 + x - 2 = 56$$

Using our 2-step method to solving algebraic equations, we can solve for x:

$$\begin{aligned}
x + 4 + x + 4 + x - 2 + x - 2 &= 56 \\
4x + 4 &= 56 \\
4x + 4 \ -4 &= 56 \ -4 \\
4x &= 52 \\
\frac{4x}{4} &= \frac{52}{4} \\
x &= 13
\end{aligned}$$

Therefore, $length = x + 4 = 13 + 4 = 17$ and $width = x - 2 = 13 - 2 = 11$.
Thus, the area of the rectangle is $Area = length \times width = (17)(11) = 187$ square units.

15. Replacing a triangle with a square on one side of the balance requires the addition of a triangle on the other side to maintain balance. This implies that one square is balanced by two triangles. So the answer is c.
But how do we show this using algebra?

The key idea is to treat each of the symbols as variables (you can use a letter if you want) and to understand each balance as an equation.
Using the symbols themselves as variables, we are given the following two equations:

$$2\square + 2\triangle = \heartsuit \tag{1}$$

$$3\square + \triangle = \heartsuit + \triangle \tag{2}$$

Note that that (1) gives an expression for $\heartsuit$ in terms of $\square$ and $\triangle$.
So if we write this expression in place of $\heartsuit$ in (2), we get an equation only involving $\square$ and $\triangle$:

$$3\square + \triangle = 2\square + 2\triangle + \triangle$$

We can simplify this equation and isolate for $\square$ in terms of $\triangle$. We should get $\square = 2\triangle$ as expected.

$$\begin{aligned}
3\square + \triangle &= 2\square + 2\triangle + \triangle \\
3\square + \triangle &= 2\square + 3\triangle \\
3\square \color{red}{-2\square} + \triangle &= 2\square \color{red}{-2\square} + 3\triangle \\
\square + \triangle &= 3\triangle \\
\square + \triangle \color{red}{-\triangle} &= 3\triangle \color{red}{-\triangle} \\
\square &= 2\triangle
\end{aligned}$$

Hence, as we expected, $\triangle\triangle$ balances $\square$. So c. is the correct answer.