UNIVERSITY OF WATERLOO
Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Enriching Mathematics and Computer Science for 50 years

Grade 6 Math Circles

November 26/27, 2013

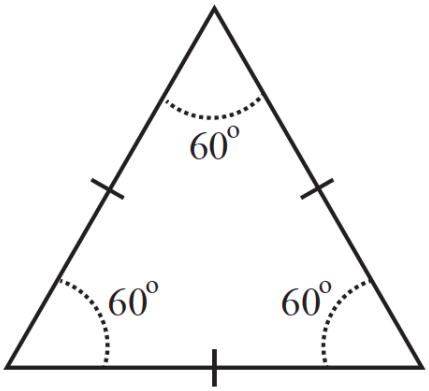
Origami & Math Trivia

Origami

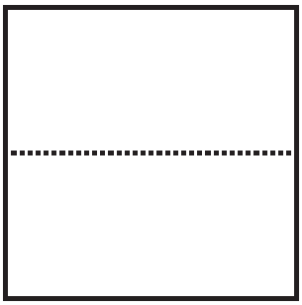
Origami is the traditional Japanese art of paper folding. The goal of origami is to take a square piece of paper and transform it using only folds and creases. There is no cutting or gluing in strict origami.

Equilateral Triangle

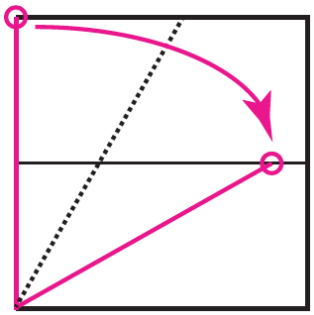
Can you fold a perfect equilateral triangle out of a square piece of paper? Just as a reminder, an equilateral triangle has equal side lengths and equal interior angles.



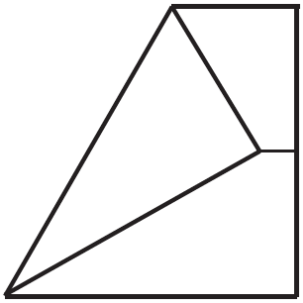
To begin, fold the paper in half and then open it again. You should have a crease dividing the paper in half.



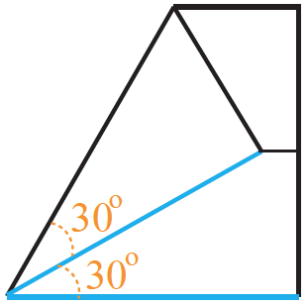
Make the upper left corner meet the middle crease in such a way that the resulting crease runs straight from the lower left corner to the top edge.



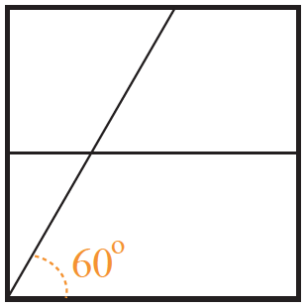
This is the result of the previous fold.



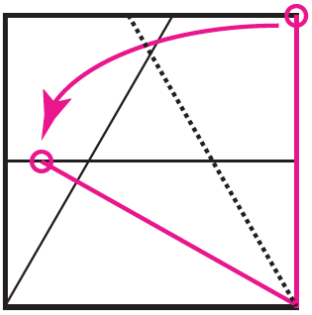
This fold has divided the lower left corner of the paper perfectly into thirds. Because the corner originally had 90°, each third must have $\frac{90}{3} = 30°$.

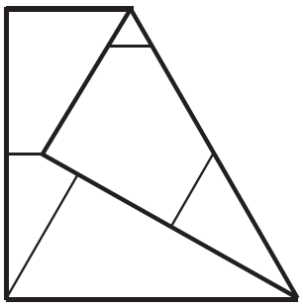


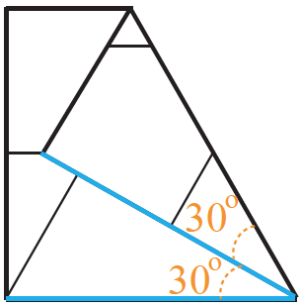
Open the paper so that it is laying flat. The crease forms a 60° angle with the bottom of the square. This is one of the three 60° angles of our equilateral triangle.



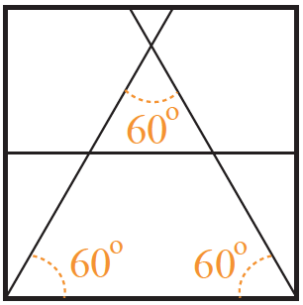
Now we repeat all of the previous steps, but with the upper right corner instead.



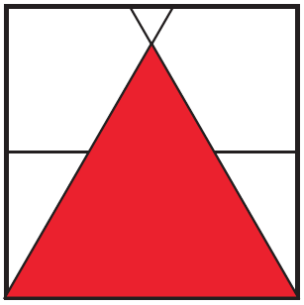




Opening the final fold results in another 60° angle in the lower right corner. Because the interior angles of a triangle must add up to 180°, the final angle at the top must be $180 - 60 - 60 = 60°$.

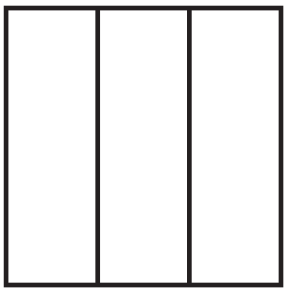


Thus we have succeeded in folding a perfect equilateral triangle out of a square piece of paper!

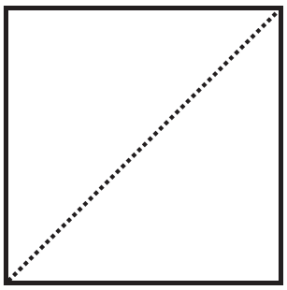


Thirds

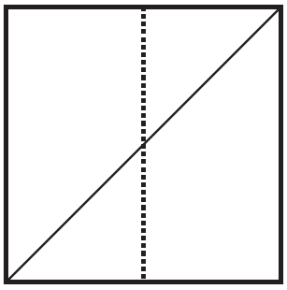
Our next task is to fold a square piece of paper into thirds.



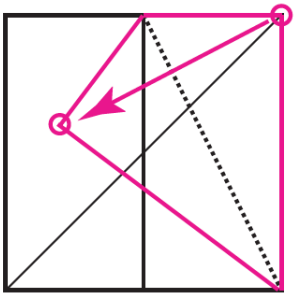
Begin by folding the paper in half diagonally, with the crease running from the lower left corner to the upper right corner.



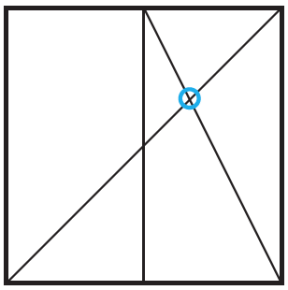
Fold it in half again, with the crease running vertically.



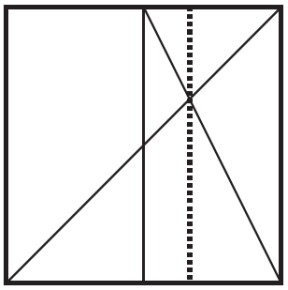
Fold the upper right corner in such a way that the resulting crease runs from the center of the paper at the top to the lower right corner.



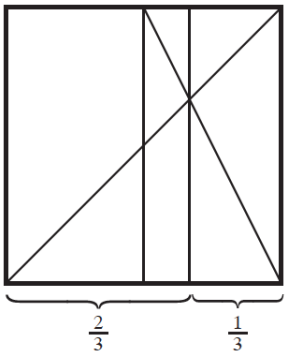
Identify the intersection of the two diagonal lines.



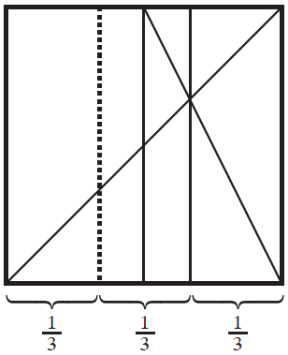
Create a vertical crease running through the point of intersection.



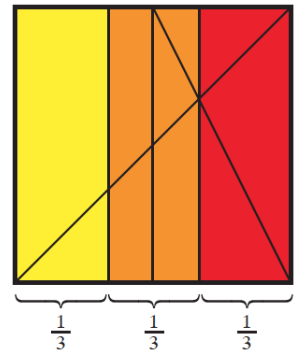
This vertical crease has divided the paper. To the left of the crease is two thirds of the paper, and to the right of the crease is one third of the paper.



Mirror this crease to the other side of the paper.



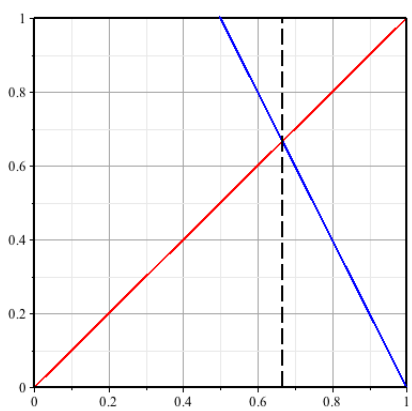
Thus we have succeeded in folding a square piece of paper into thirds!



Why does this work?

Imagine that the piece of origami paper is a graph. The bottom of the paper is the axis for the x values from 0 to 1, and the left side is the axis for the y values from 0 to 1.

The first crease you made can be represented by the equation $y = x$ (red line below), and the third crease you made can be represented by the equation $y = -2x + 2$ (blue line below).

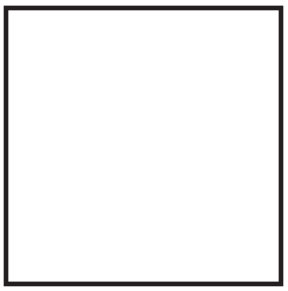


By solving the system of equations, or simply by observing the point of intersection, it is easy to see that the lines intersect only when $x = \dfrac{2}{3}$ or $x = 0.6\overline{6}...$

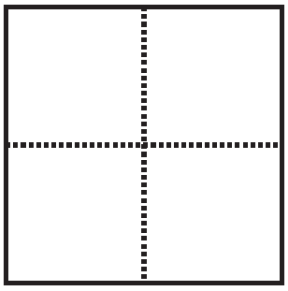
Heart

Time for some real origami!

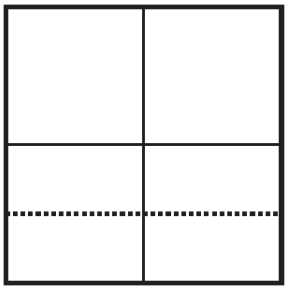
Begin with a square piece of paper. If you have real origami paper that is coloured on one side, start with the white side facing you.

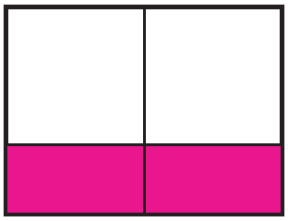


Fold the paper in half both vertically and horizontally. Then lay the paper flat again.

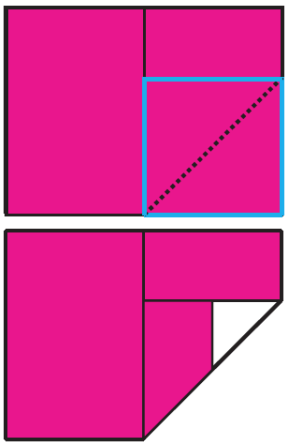


Fold the bottom edge to meet the horizontal middle crease.

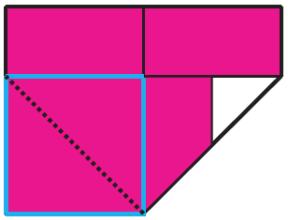


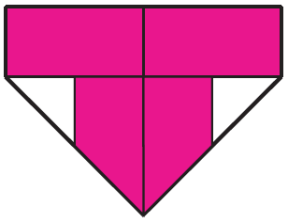


Flip the paper over. Now fold the lower right corner so that the bottom edge meets the vertical crease.

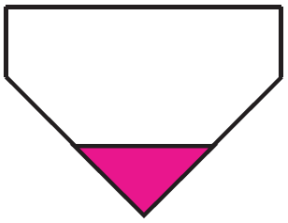


Repeat on the other side. Fold the lower left corner so that the bottom edge meets the vertical crease.

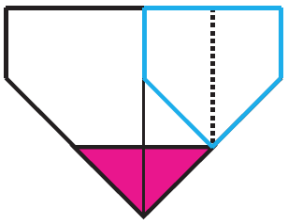


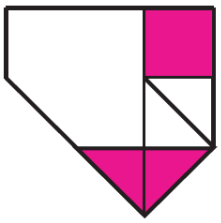


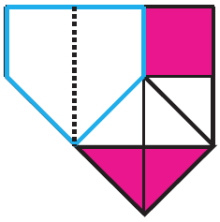
Flip the paper over.



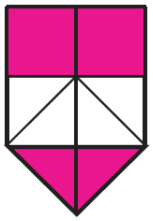
Fold the right edge to meet the vertical crease.



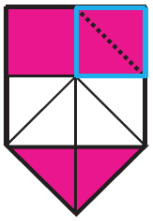




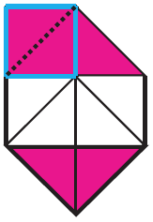
Repeat on the other side. Fold the left edge to meet the vertical crease.



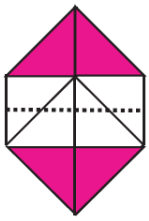
Fold the upper right corner so that the top edge meets the vertical crease.



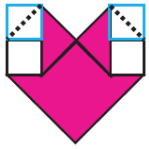
Repeat on the other side. Fold the upper left corner so that the top edge meets the vertical crease.



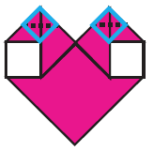
Fold the paper in half horizontally so that the top edge meets the bottom edge. At the same time, flatten out the two paper pockets that appear on the other side (this is the tricky part!).



Fold down both upper corners so that the top edges meet at the center crease.



Fold down the points. You can also slide the front flap into the pocket found on the inside of the back flap.



Flip the paper over. You have just created an origami heart!

Math Trivia

We ended Fall 2013 Math Circles with a game of Math Trivia, in the style of Jeopardy, to review the topics we had covered over the previous seven weeks. If you would like the PowerPoint presentation we used, feel free to email me at akokic@uwaterloo.ca.

Binary and Beyond

Value	Question
100	What are the allowed digits in the base 8 number system?
200	What is the decimal value of the binary number 11001?
300	Order the following binary numbers from least to greatest: 110, 11, 1000, 101, 100
400	What is the decimal value of the base 3 number 12012?
500	Add the binary numbers 10010 + 11111.

History of Cryptography

Value	Question
100	If you encrypt plaintext, what is the result?
200	Where was Bletchley Park?
300	Which two letters are often combined in the Polybius Square cipher?
400	What is a product cipher?
500	Why is the Atbash cipher a very weak cipher?

Divisibility

Value	Question
100	Is 10648 divisible by 2?
200	Is 632123 divisible by 3?
300	Is 1092762 divisible by 9?
400	What is the prime factorization of 100?
500	Is 8543210 divisible by 6?

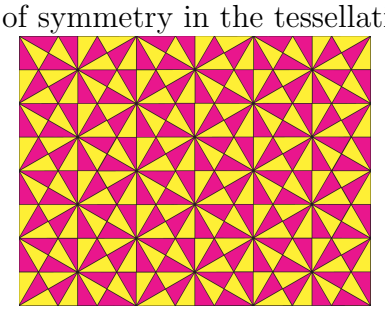
Dimensional and Unit Analysis

Value	Question
100	Name 4 of the 7 base dimensions.
200	What is the dimension of acceleration?
300	Is the formula $\frac{4}{3}\pi r^3$ (where r is the radius) the surface area or volume of a sphere?
400	40 hA = _____ mA
500	Does the equation $x + y = z$ (where $[x]$=M, $[y]$=L, and $[z]$=M) obey rules for dimensional quantities?

Algebra

Value	Question
100	What does BEDMAS stand for?
200	Evaluate: $11 - 6^2 \div 12 + (4 + 5 \times 2) \div 7$
300	Use FOIL to expand $(a + b)(a - b)$.
400	Solve for x: $3x - 5 = 1$
500	If $2x + 7 = 17$, what is $5x$?

Smorgasbord

Value	Question
100	At which universities did Bill Tutte teach?
200	Identify all the kinds of symmetry in the tessellation below. 
300	What is the GCF of 50 and 275?
400	When do you stop making the table for the Russian Peasant method?
500	If $[x]$=L, $[m]$=M, and $[t]$=T, and $xm = bt$, then what is $[b]$?