

Grade 7 & 8 Math Circles

October 29/30, 2013

Logic Puzzles

Introduction

Mathematics isn't at all about memorizing formulas or doing procedures over and over. It's all about thinking logically, finding patterns and connections, and solving problems.

A logic puzzle is a problem, challenge, or game that requires the player to use forms of critical thinking to arrive at a solution.

Strategies

Some tips to keep in mind when solving logic puzzles:

- Read and reread the problem until you fully understand it and its goals.
- Organize your information in a chart or diagram to focus only on relevant points.
- Use logical reasoning to eliminate options.
- Tackle simpler sub problems, but keep the big picture in mind.
- List out all of the possibilities if you can, and use “guess and check”.
- Take it one step at a time.
- Use a puzzle's rules and guidelines to double-check your work.
- Be persistent. If you become stuck, remember that these problems are meant to be fun!

The most important thing to remember when working on logic puzzles is that they are *logical*; every step has to make sense and be verifiable.

Sudoku

The goal when filling out a sudoku is to enter a number from 1 to 9 in each box of the puzzle. Each row, column, and outlined 3×3 region must contain each number only once.

Example I

Sudoku Puzzle - Easy

9	4	1	8					2
	6	8	4	5	2			3
		5		6	9		4	8
6			5		7	8		4
8	5	7		2	4	9	3	1
2		4		3	8	5	6	
4		6	7	9	1	2		
1		9	2		5		7	6
		2				4	1	9

www.sudoku-puzzles.net

2D-Sudoku

Fill every row, columns, and shaded diagonal with the numbers from 1 to 5.

Example II

1				
3				4
	4		2	
			3	

Minesweeper

Draw a mine in some cells of the grid. The number in a cell indicates how many of the eight neighbouring cells contain a mine. A numbered cell does not contain a mine.

Example III

2				
4			3	
			0	
4	5	4		1

There are many more fun number grid puzzles in the problem set!

Word Problems

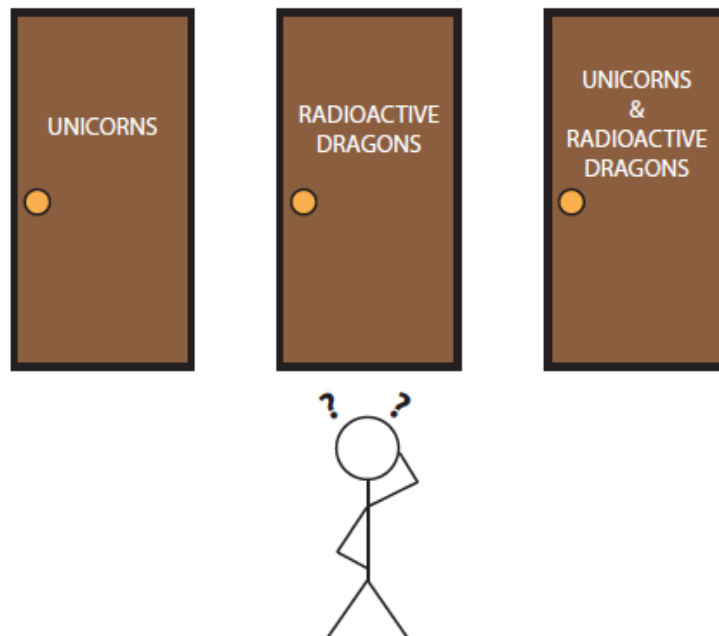
Every word problem is different, so there are also many different ways to solve them; use your intuition and consider all the possible cases, eliminating contradictions and impossible cases.

Example IV

What is the four-digit number in which the first digit is one-third the second, the third is the sum of the first and second, and the last is three times the second?

Example V

Three rooms have doors that are labelled “Unicorns”, “Radioactive Dragons”, and “Unicorns and Radioactive Dragons”. You know that each door is incorrectly labelled. You are allowed to peek inside only one of the rooms exactly once, and then you have to make a decision. Describe a **sure-fire** way to figure out which room is which.



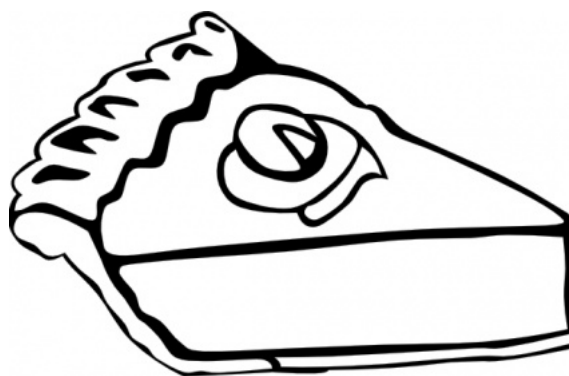
Example VI

Every Halloween, the Winchester family hosts their annual Halloween Pumpkin Pie Bake Off. However, this year a rotten thief has stolen and hidden away the prized pumpkin pie!

Detectives come and narrow it down to five suspects: Bobby, Sam, Dean, Ellen, and Jo.

Under questioning, each suspect makes two statements. Using police intuition, the detectives realize for each suspect, exactly one of their statements is true, and one of their statements is a lie.

- Dean: It was Sam.
It wasn't Ellen.
- Jo: It was Dean.
It was Bobby.
- Sam: It was Jo.
It wasn't Ellen.
- Ellen: It wasn't Sam.
It was Bobby.
- Bobby: It wasn't Dean.
It wasn't Sam.



Who is the pie thief?

Example VII

A census taker approaches a woman leaning on her gate and asks about her children. She says, "I have three children and the product of their ages is thirty-six. The sum of their ages is the number on this gate." The census taker does some calculation and claims not to have enough information. The woman enters her house, but before slamming the door tells the census taker, "I have to see to my eldest child who is in bed with measles." The census taker departs, satisfied.

What are the ages of the three children?

Logic-Grid Puzzle

As the name suggests, these puzzles use grids to help keep you organized. This is very helpful as logic-grid puzzles can get very complicated.

Example VIII

Five sisters all have their birthday in a different month and each on a different day of the week. Using the clues below and the grid, determine the month and day of the week each sister's birthday falls.

1. Paula was born in March but not on Saturday. Abigail's birthday was not on Friday or Wednesday.
2. The girl whose birthday is on Monday was born earlier in the year than Brenda and Mary.
3. Tara wasn't born in February and her birthday was on the weekend.
4. Mary was not born in December nor was her birthday on a weekday. The girl whose birthday was in June was born on Sunday.
5. Tara was born before Brenda, whose birthday wasn't on Friday. Mary wasn't born in July.

(Source: puzzlersparadise.com)

	February	March	June	July	December	Sunday	Monday	Wednesday	Friday	Saturday
Abigail										
Brenda										
Mary										
Paula										
Tara										
Sunday										
Monday										
Wednesday										
Friday										
Saturday										

Problem Set

1. SUDOKU.

(a)

Sudoku Puzzle - Medium

			6	9		7		
					4		3	
7						9	2	6
5				2	3			8
		6			9		5	2
		8	1					3
	4		8	5	7	3	6	
8	6		3					
				4			8	1

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(b)

Sudoku Puzzle - Hard

			2			1		
	7	1			9			2
				1	4		9	
	9	2				3		
3					6	7		
					3		2	
5	6				7			9
				4			5	3
	1							

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3. **HIDATO.** The goal of Hidato is to fill the grid with numbers such that consecutive numbers connect horizontally, vertically, or diagonally. The highest and smallest numbers are indicated and some numbers are already printed on the grid.

(Source: mathinenglish.com)

(a)

1		4	
2	13		7
		8	
16	15		10

(b)

				24	
18	16	20	23		
		12	29	30	27
14			5		31
	7	36			3
9			34	1	

4. **ARUKONE**. Make a line between the matching numbers. The lines can only be horizontal and vertical, and must be continuous. No lines should cross or overlap. There should be no empty cells in your solution.

(Source: menneske.no)

6	5							
				4			7	
3								
							7	
		5					1	
	6				3			
					2			
4				1				2

5. **SHIKAKU**. Shikaku is played on a rectangular grid. Some of the squares in the grid are numbered. The objective is to divide the grid into rectangular and square pieces such that each piece contains exactly one number, and that number represents the area of the rectangle.

(Source: nikoli.com)

	5						10	4						4
	3		2							2	2			
		4		9						9	6			
				4		2							6	6
						6							2	4
	3	8						3						
	4	4						4		4				
				4	2					6			6	
				4	2								6	6
8							6	8						2

6. **2D-SUDOKU.** Fill every row, columns, and shaded diagonal with the numbers from 1 to 7.

		6	2	4		
	6				7	
3						
1			4	6		
4					6	
	3				2	
		2	3	7		

7. **MINESWEEPER.** Shade the cells that contain mines. The number in a cell specifies how many mines that cell is touching. Cells with numbers never contain mines.

1			2			1	
		2			3		
2				2			2
		3		2			
1					1		2
1			2		2	2	
1		2					
	1			2		1	

Word Problems

8. At a restaurant downtown, Mr. Red, Mr. Blue, and Mr. White meet for lunch. Under their coats they are wearing either a red, blue, or white shirt.

Mr. Blue says, “Hey, did you notice we are all wearing different coloured shirts from our names?”

The man wearing the white shirt says, “Wow, Mr. Blue, that’s right.”

Can you tell who is wearing what colour shirt?

9. You are an archaeologist that has just unearthed a long-sought pair of ancient treasure chests. One chest is plated with silver, and the other is plated with gold. According to legend, one of the two chests is filled with great treasure, whereas the other chest houses a man-eating python. Faced with a dilemma, you then notice that there are inscriptions on the chests:

- **Silver Chest:** This chest contains the python
- **Gold Chest:** Exactly one of these two inscriptions is true.

Based on these inscriptions, which chest contains the treasure?

10. Alex, Bret, Chris, Derek, Eddie, Fred, Greg, Harold, and John are nine students who live in a three storey building, with three rooms on each floor. A room in the West wing, one in the centre, and one in the East wing. If you look directly at the building, the left side is West and the right side is East. Each student is assigned exactly one room. Can you find where each of their rooms is:

- Harold does not live on the bottom floor.
- Fred lives directly above John and directly next to Bret.
- Eddie lives to the right of the room that is directly above Fred.
- Derek lives directly above Fred.
- Greg lives directly above Chris.

11. There is a secret hall at Hogwart’s with magical closets. When you open a closet door, you might find Harry Potter, or you might find Voldemort. Since Hogwarts is magic, it is possible that both doors contain Harry Potter, or both doors contain Voldemort.

- (a) When you enter the secret hall, you encounter two closet doors. “One of the signs is true,” says Professor Snape, “and the other is false.”

- i. **First Door:** Harry is behind this door. Voldemort is behind the other.
- ii. **Second Door:** Harry is behind one of these doors. Voldemort is behind the other.

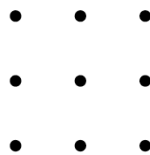
Who is behind each door?

- (b) You enter another secret hall and encounter two more closet doors. This time, Snape says that either both doors are true or both doors are false.
 - i. **First Door:** Harry is behind at least one of these doors.
 - ii. **Second Door:** Voldemort is behind the other door.

Who is behind each door?

Thinking Outside the Box Problems

- 12. Two fathers took their sons fishing. Each person caught one fish, but when they returned to camp, there were only 3 fish. How is this possible?
- 13. You have two **unmarked** buckets - one that can hold 3 L, and another that can hold 5 L. How can you measure 4 L using only these two buckets?
- 14. You have two hourglasses. One takes 7 minutes to fully drain, another takes 11 minutes. Using only these two hourglasses, how can you time exactly 15 minutes?
- 15. A farmer has 10 trees. He plants them in 5 rows of 4 trees. How is this possible?
- 16. Without lifting your pencil or retracing any lines, can you connect all 9 dots below by drawing only 4 straight lines? (Hint: Think outside the box!)



Harder Problems

17. * The King called the three wisest men in the country to his court to decide who would become his new advisor. He placed a hat on each of their heads, such that each wise man could see all of the other hats, but none of them could see their own. Each hat was either white or blue. The king gave his word to the wise men that at least one of them was wearing a blue hat - in other words, there could be one, two, or three blue hats, but not zero. The king also announced that the contest would be fair to all three men. The wise men were also forbidden to speak to each other. The king declared that whichever man stood up first and announced the color of his own hat would become his new advisor. The wise men sat for a very long time before one stood up and correctly announced the answer. What did he say, and how did he work it out?
18. After solving the previous chest problem and winning the treasure, you get lucky again! This time you and three ancient treasure chests. One chest is plated with silver, one with gold, and one with bronze. According to legend, one of the three chests is filled with great treasure, whereas the other two chests both house man-eating pythons. Faced with a dilemma, you then notice that there are inscriptions on the chests:
- **Silver Chest:** The treasure is in this chest.
 - **Gold Chest:** Treasure is not in this chest.
 - **Bronze Chest:** Treasure is not in the Gold Chest.

Which chest contains the treasure?

19. ** There is a hallway with 50 light switches, numbered 1 to 50. Each of them is initially off.
- You start at one end of the hallway. On your **first** pass, you turn on every switch. When you reach the end of the hallway, you return back to the beginning of the hallway.
- On your second pass, you go to every second switch (so you only look at the ones marked 2, 4, 6, 8, 10, 12, ..., 50). If the switch is off, you turn it on. If the switch is on, you turn it off. Once you reach the end of the hallway, you return to the beginning.
- On your third pass, you go to every third switch (3, 6, 9, 12, 15, ..., 48). If the switch is off, you turn it on. If the switch is on, you turn it off. Once you reach the end of the hallway, you return to the beginning.
- You repeat this process until you have made 50 passes. At the end of it all, which light switch(es) are still on?

20. * Last night it snowed! Harry and his friends met up with the other kids at the park for the first snowball fight of the season. Since Harry and his friends were on different teams, they only managed to get one chance to hit their friends with snowballs. Several hours later, all the boys were very wet, tired and hungry. They headed back to Harry's home for a drink and a snack, his mom's fresh baked cookies. Determine the full name of each boy, who was hit by who, what each boy had to drink, and what kind of cookie each chose.
- (a) Barry, whose last name wasn't Horn, hit Nathaniel but he got hit by the boy who had the oatmeal raisin cookies. The boy who got peanut butter cookies didn't hit Curtis.
 - (b) The snowball that hit Barry wasn't thrown by Nathaniel, whose last name wasn't West. Randy Mann didn't have milk with his cookies. The boy whose last name was Jones didn't have chocolate chip cookies.
 - (c) Randy had neither water to drink nor chocolate in his cookies. The boy whose last name was Horn had neither sugar cookies nor apple juice. The boy who hit Harry had chocolate cookies.
 - (d) The boys who hit Nathaniel and Curtis had apple juice and orange juice to drink, not necessarily in that order. Harry, whose last name wasn't Horn, didn't hit Barry nor was he hit by Nathaniel.
 - (e) The boys who had chocolate in their cookies didn't have Hathaway for a last name. The boy who had peanut butter cookies had juice to drink, but his last name wasn't Jones.
 - (f) The boy who hit Randy had West for a last name. The boy who had iced tea didn't hit Harry. The Jones boy had water to drink.

		Last Name					Target					Beverage					Cookie				
Thrower	Barry																				
	Curtis																				
	Harry																				
	Nathaniel																				
	Randy																				
Cookie	chocolate																				
	chocolate chip																				
	oatmeal raisin																				
	peanut butter																				
	sugar																				
Beverage	apple juice																				
	iced tea																				
	milk																				
	orange juice																				
	water																				
Target	Barry																				
	Curtis																				
	Harry																				
	Nathaniel																				
	Randy																				