UNIVERSITY OF WATERLOO
Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Enriching Mathematics and Computer Science for 50 years

Grade 7 & 8 Math Circles

November 26/27/28, 2013

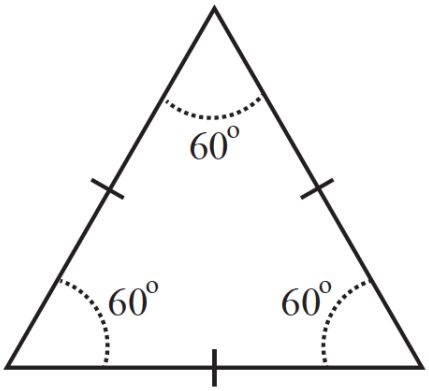
Origami & Math Trivia

Origami

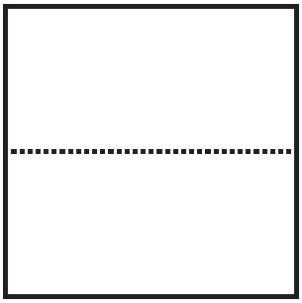
Origami is the traditional Japanese art of paper folding. The goal of origami is to take a square piece of paper and transform it using only folds and creases. There is no cutting or gluing in strict origami.

Equilateral Triangle

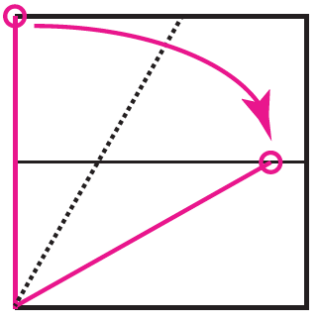
Can you fold a perfect equilateral triangle out of a square piece of paper? Just as a reminder, an equilateral triangle has equal side lengths and equal interior angles.



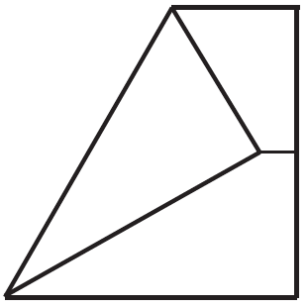
To begin, fold the paper in half and then open it again. You should have a crease dividing the paper in half.



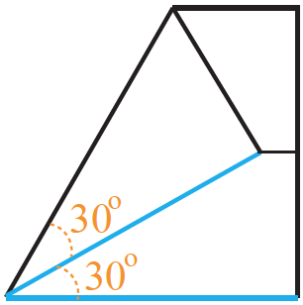
Make the upper left corner meet the middle crease in such a way that the resulting crease runs straight from the lower left corner to the top edge.



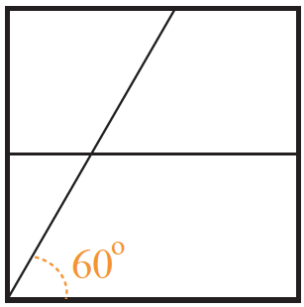
This is the result of the previous fold.



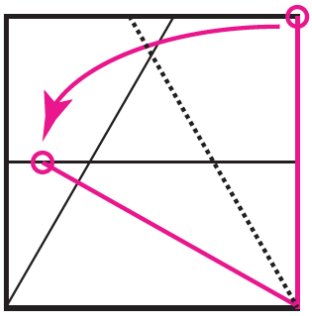
This fold has divided the lower left corner of the paper perfectly into thirds. Because the corner originally had 90°, each third must have $\frac{90}{3} = 30°$.

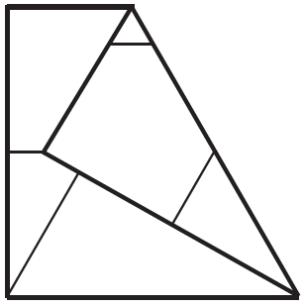


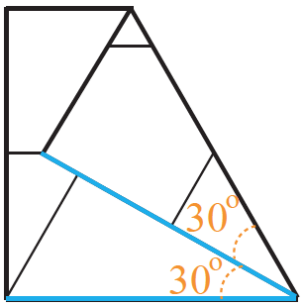
Open the paper so that it is laying flat. The crease forms a 60° angle with the bottom of the square. This is one of the three 60° angles of our equilateral triangle.



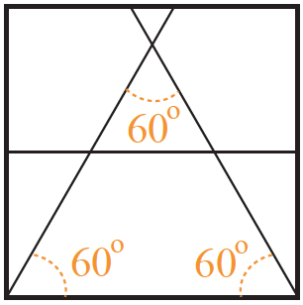
Now we repeat all of the previous steps, but with the upper right corner instead.

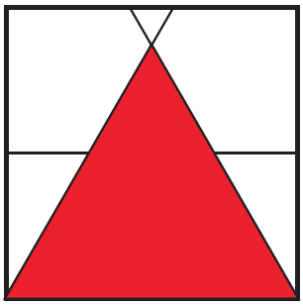






Opening the final fold results in another 60° angle in the lower right corner. Because the interior angles of a triangle must add up to 180°, the final angle at the top must be $180 - 60 - 60 = 60°$.

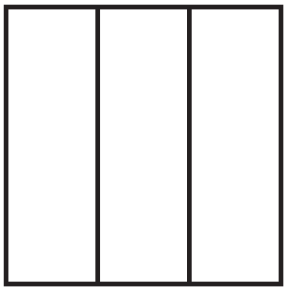




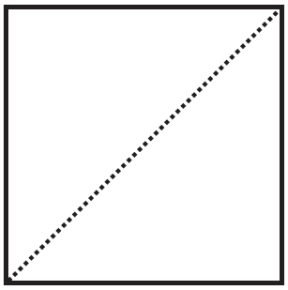
Thus we have succeeded in folding a perfect equilateral triangle out of a square piece of paper!

Thirds

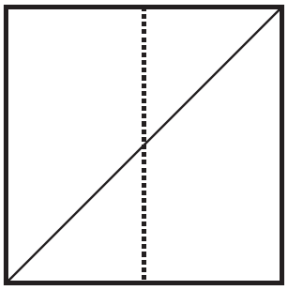
Our next task is to fold a square piece of paper into thirds.



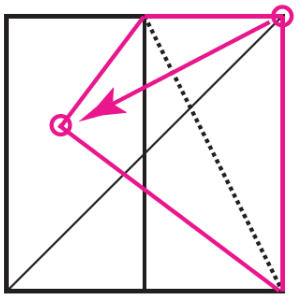
Begin by folding the paper in half diagonally, with the crease running from the lower left corner to the upper right corner.



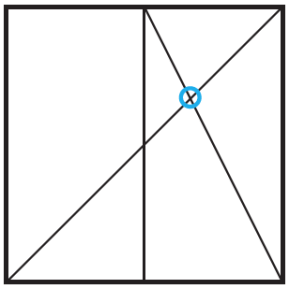
Fold it in half again, with the crease running vertically.



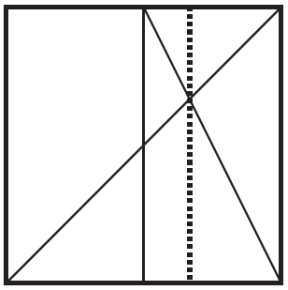
Fold the upper right corner in such a way that the resulting crease runs from the center of the paper at the top to the lower right corner.



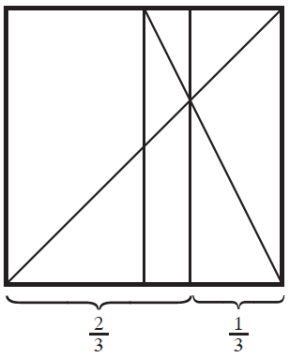
Identify the intersection of the two diagonal lines.



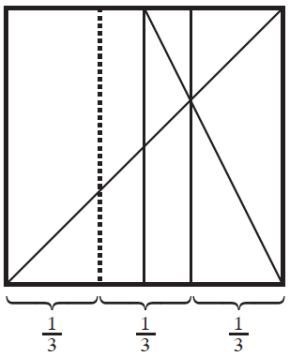
Create a vertical crease running through the point of intersection.



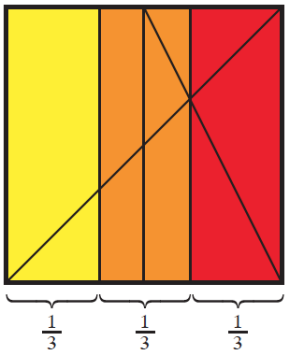
This vertical crease has divided the paper. To the left of the crease is two thirds of the paper, and to the right of the crease is one third of the paper.



Mirror this crease to the other side of the paper.



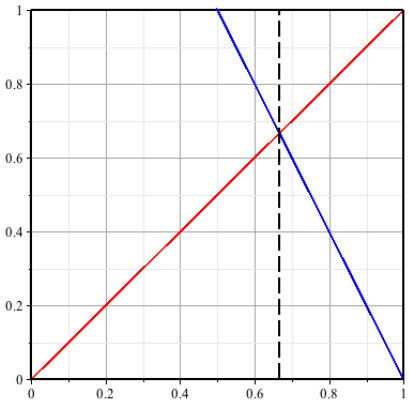
Thus we have succeeded in folding a square piece of paper into thirds!



Why does this work?

Imagine that the piece of origami paper is a graph. The bottom of the paper is the axis for the x values from 0 to 1, and the left side is the axis for the y values from 0 to 1.

The first crease you made can be represented by the equation $y = x$ (red line below), and the third crease you made can be represented by the equation $y = -2x + 2$ (blue line below).

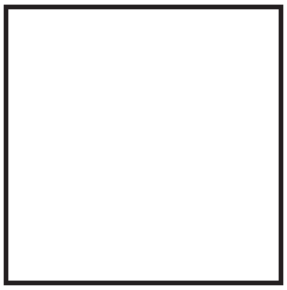


By solving the system of equations, or simply by observing the point of intersection, it is easy to see that the lines intersect only when $x = \frac{2}{3}$ or $x = 0.6\overline{6}...$

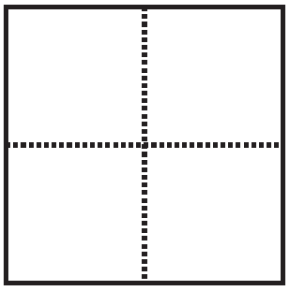
Heart

Time for some real origami!

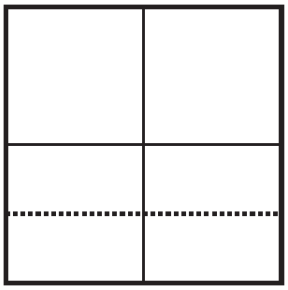
Begin with a square piece of paper. If you have real origami paper that is coloured on one side, start with the white side facing you.



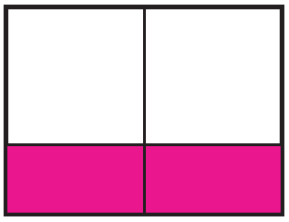
Fold the paper in half both vertically and horizontally. Then lay the paper flat again.

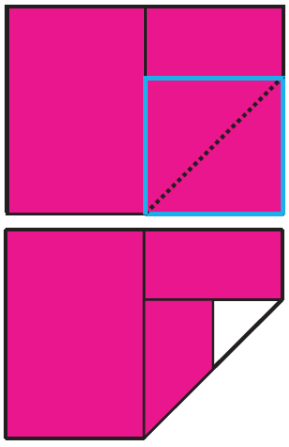


Fold the bottom edge to meet the horizontal middle crease.

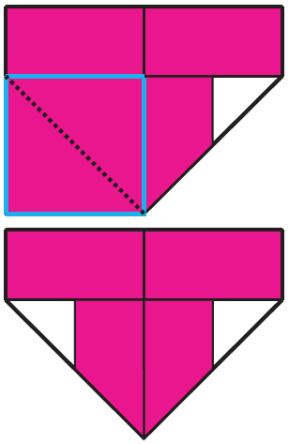


Flip the paper over. Now fold the lower right corner so that the bottom edge meets the vertical crease.

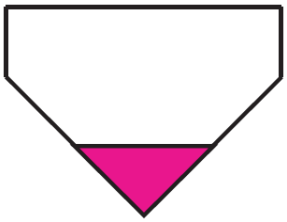




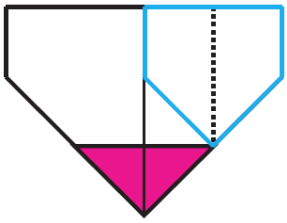
Repeat on the other side. Fold the lower left corner so that the bottom edge meets the vertical crease.

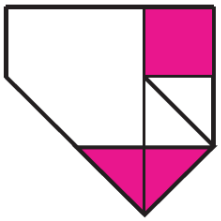


Flip the paper over.

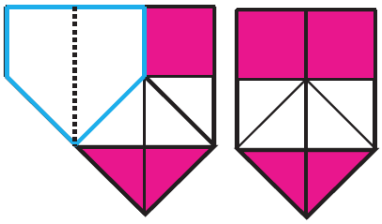


Fold the right edge to meet the vertical crease.

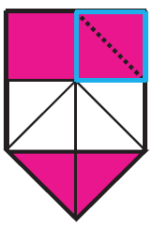




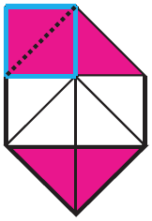
Repeat on the other side. Fold the left edge to meet the vertical crease.



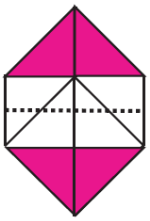
Fold the upper right corner so that the top edge meets the vertical crease.



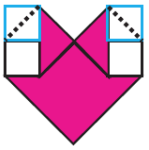
Repeat on the other side. Fold the upper left corner so that the top edge meets the vertical crease.



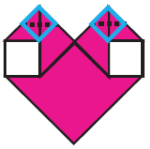
Fold the paper in half horizontally so that the top edge meets the bottom edge. At the same time, flatten out the two paper pockets that appear on the other side (this is the tricky part!).



Fold down both upper corners so that the top edges meet at the center crease.



Fold down the points. You can also slide the front flap into the pocket found on the inside of the back flap.



Flip the paper over. You have just created an origami heart!

Math Trivia

We ended Fall 2013 Math Circles with a game of Math Trivia, in the style of Jeopardy, to review the topics we had covered over the previous seven weeks. If you would like the PowerPoint presentation we used, feel free to email me at adam.rodrigues@uwaterloo.ca.

In class, no calculators or notes were used to help solve the trivia questions. Using them as aids is up to you, but I highly encourage you to try and answer these questions without your calculator or notes.

Algebra

Value	Question
100	Evaluate $11 - 6^2 \div 12 + (4 + 5 \times 2) \div 7$
200	Expand $(a + b)(a - b)$
300	Solve for x: $3x - 5 = 1$
400	If $2x + 7 = 17$, what is $5x$?
500	Solve the following system of equations: $x = 4y - 3$ $y + 2x = 12$

History of Numbers

Value	Question
100	Give three examples of integers.
200	Give three examples of irrational numbers.
300	What happened to the man who discovered irrational numbers?
400	What is the complex conjugate of $1 + 3i$?
500	Evaluate $(4 + 3i) + (6 - 2i)$

Finance

Value	Question
100	Calculate 15% of 300.
200	What is a reward for lending money or a price for borrowing money called?
300	If interest is compounded weekly, what is m?
400	What is the simple interest due on a \$1000 loan at the end of 1 year if the annual interest rate is 5%?
500	I deposit \$1000 into a savings account that earns 10% simple interest annually. How much money will be in the account in 5 years?

Logic Puzzles

Value	Question
100	Sarahs mom has four kids: One daughter named April, one daughter named May, and one daughter named June. What is the name of the fourth child?
200	The room you are in is pitch black. You know that there are 6 white socks, 10 black socks, and 16 blue socks in a drawer. How many socks do you need to pull out of the drawer to be certain that you have at least one matching pair?
300	Two fathers took their sons fishing. Each person caught one fish, but when they returned to camp, there were only 3 fish. How is this possible?
400	A ladder hangs over the side of a ship anchored in a port. The bottom rung touches the water. The distance between rungs is 20cm and the length of the ladder is 180cm. The tide is rising at a rate of 20cm each hour. How many rungs will touch the water in one hour?
500	What is the four-digit number in which the first digit is one-quarter the second, the third is the sum of the first and second, and the last is two times the second?

Similarity and Congruence

Value	Question
100	If two triangles are congruent, are they similar as well?
200	Are all regular hexagons similar to each other?
300	What are the differences between the similarity tests SAS and SSRA (or HL)?
400	If a polygon is scaled by a factor f, then by what factor is its area scaled by?
500	Why cant SSA be a similarity test in general? Name the special type of case that causes trouble.

Mathematical Gems

Value	Question
100	What famous mathematician derived the formula for the sum of the first n natural numbers?
200	What is the common ratio of this geometric series? $4 + 12 + 36 + 108 + \ldots$
300	What is the sum of the first 300 odd natural numbers?
400	A right-angled triangle has a hypotenuse of 26 cm and one side of 10 cm. What is the length of the other side?
500	What is the sum of the natural numbers from 20 to 80 inclusive?

Tie-breaker!

A domino covers 2 adjacent squares on a chess board. If the corner squares on opposite sides of an 8 x 8 board are removed, can the board be tessellated by dominoes? Why or why not?