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Grade 7 & 8 Math Circles

October 22/23, 2013

Finance Solutions

Problem Set

1. (a) $S = ?$ $\quad P = 1200$ $\quad r = 0.05$ $\quad t = \frac{7}{12}$

$$\begin{aligned} S &= P(1 + rt) \\ &= 1200\left(1 + (0.05)\left(\frac{7}{12}\right)\right) \\ &= 1235 \end{aligned}$$

The accumulated value is \$1235.

(b) $S = ?$ $\quad P = 8000$ $\quad r = 0.125$ $\quad t = 4$

$$\begin{aligned} S &= P(1 + rt) \\ &= 8000(1 + (0.125)(4)) \\ &= 12000 \end{aligned}$$

The accumulated value is \$12000.

(c) $S = ?$ $\quad P = 500$ $\quad r = 0.1$ $\quad t = \frac{99}{365}$

$$\begin{aligned} S &= P(1 + rt) \\ &= 500\left(1 + (0.1)\left(\frac{99}{365}\right)\right) \\ &= 513.56 \end{aligned}$$

The accumulated value is \$513.56.

(d) $S = ?$ $P = 750$ $r = 0.1325$ $t = \frac{15}{52}$

$$\begin{aligned}S &= P(1+rt)\\ &= 750\left(1+(0.1325)\left(\frac{15}{52}\right)\right)\\ &= 778.67\end{aligned}$$

The accumulated value is \$778.67.

2. $S = 3600$ $P = ?$ $r = 0.08$ $t = 10$

$$\begin{aligned}S &= P(1+rt)\\ P &= \frac{S}{1+rt}\\ &= \frac{3600}{1+(0.08)(10)}\\ &= 2000\end{aligned}$$

The principal/present value is \$2000.

3. $P = 100$ $I = 20$ $r = ?$ $t = \frac{10}{12}$

$$\begin{aligned}I &= Prt\\ \frac{I}{\color{red}{Pt}} &= \frac{Prt}{\color{red}{Pt}}\\ r &= \frac{I}{Pt}\\ &= \frac{20}{(100)\left(\frac{10}{12}\right)}\\ &= 0.24\end{aligned}$$

The annual simple interest rate is 24%.

Using the other equation:

$P = 100 \qquad S = 120 \qquad r = ? \qquad t = \frac{10}{12}$

$$\begin{aligned}
S &= P(1 + rt) \\
\frac{S}{P} &= \frac{P(1 + rt)}{P} \\
\frac{S}{P} - 1 &= 1 + rt - 1 \\
\frac{\frac{S}{P} - 1}{t} &= \frac{rt}{t} \\
r &= \frac{\frac{S}{P} - 1}{t} \\
&= \frac{\frac{120}{100} - 1}{\frac{10}{12}} \\
&= 0.24
\end{aligned}$$

The annual simple interest rate is 24%.

4. $P = 3000 \qquad I = 60 \qquad r = 0.06 \qquad t = ?$

$$\begin{aligned}
I &= Prt \\
\frac{I}{Pr} &= \frac{Prt}{Pr} \\
t &= \frac{I}{Pr} \\
&= \frac{60}{(3000)(0.06)} \\
&= 0.333... \\
&= \frac{1}{3}
\end{aligned}$$

It will take $\frac{1}{3}$ of a year (4 months) to earn \$60 interest.

5. (a) $S = ? \qquad P = 2000 \qquad r = 0.05 \qquad t = 4$

$$\begin{aligned}
S &= P(1 + r)^t \\
&= 2000(1 + 0.05)^4 \\
&= 2431.01
\end{aligned}$$

The accumulated value is \$2431.01.

(b) $S = ?$ $\quad P = 100$ $\quad r = 0.075$ $\quad t = 25$

$$\begin{aligned} S &= P(1+r)^t \\ &= 100(1+0.075)^{25} \\ &= 609.83 \end{aligned}$$

The accumulated value is \$609.83.

6. (a) $S = 7500$ $\quad P = ?$ $\quad r = 0.08$ $\quad t = 10$

$$\begin{aligned} S &= P(1+r)^t \\ P &= \frac{S}{(1+r)^t} \\ &= \frac{7500}{(1+0.08)^{10}} \\ &= 3473.95 \end{aligned}$$

The principal/present value is \$3473.95.

(b) $S = 25000$ $\quad P = ?$ $\quad r = 0.045$ $\quad t = 50$

$$\begin{aligned} S &= P(1+r)^t \\ P &= \frac{S}{(1+r)^t} \\ &= \frac{25000}{(1+0.045)^{50}} \\ &= 2767.74 \end{aligned}$$

The principal/present value is \$2767.74.

7. $S = 5000$ $\quad P = ?$ $\quad r = 0.05$ $\quad t = 4$

(a)

$$\begin{aligned} S &= P(1+rt) \\ P &= \frac{S}{(1+rt)} \\ &= \frac{5000}{(1+(0.05)(4))} \\ &= 4166.67 \end{aligned}$$

You must invest \$4166.67.

(b)

$$\begin{aligned} S &= P(1+r)^t \\ P &= \frac{S}{(1+r)^t} \\ &= \frac{5000}{(1+0.05)^4} \\ &= 4113.51 \end{aligned}$$

You must invest \$4113.51.

8. (a) $S = ?$ $\quad P = 1000$ $\quad m = 52$ $\quad r^{(52)} = 0.13$ $\quad n = 52 \times 3 = 156$

$$\begin{aligned} S &= P\left(1+\frac{r^{(m)}}{m}\right)^n \\ &= 1000\left(1+\frac{0.13}{52}\right)^{156} \\ &= 1476.26 \end{aligned}$$

The accumulated value is \$1476.26.

(b) $S = ?$ $\quad P = 500$ $\quad m = 2$ $\quad r^{(2)} = 0.04$ $\quad n = 2 \times 25 = 50$

$$\begin{aligned} S &= P\left(1+\frac{r^{(m)}}{m}\right)^n \\ &= 500\left(1+\frac{0.04}{2}\right)^{50} \\ &= 1345.79 \end{aligned}$$

The accumulated value is \$1345.79.

9. (a) $S = 6000 \qquad P = ? \qquad m = 4 \qquad r^{(4)} = 0.15 \qquad n = 4 \times 10 = 40$

$$S = P\left(1 + \frac{r^{(m)}}{m}\right)^n$$

$$P = \frac{S}{\left(1 + \frac{r^{(m)}}{m}\right)^n}$$

$$= \frac{6000}{\left(1 + \frac{0.15}{4}\right)^{40}}$$

$$= 1376.03$$

The principal/present value is \$1376.03.

(b) $S = 25000 \qquad P = ? \qquad m = 12 \qquad r^{(12)} = 0.12 \qquad n = 12 \times 50 = 600$

$$S = P\left(1 + \frac{r^{(m)}}{m}\right)^n$$

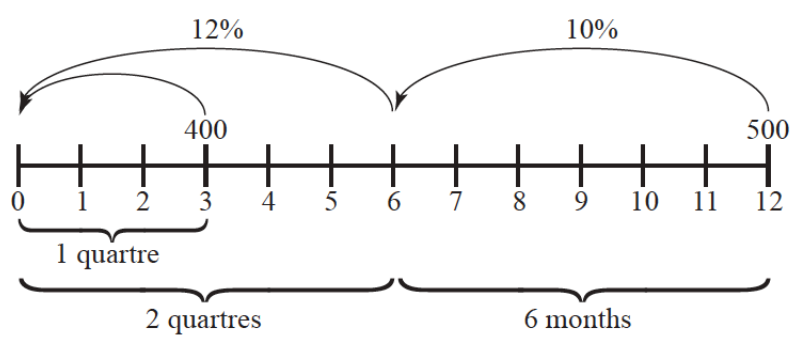
$$P = \frac{S}{\left(1 + \frac{r^{(m)}}{m}\right)^n}$$

$$= \frac{25000}{\left(1 + \frac{0.12}{12}\right)^{600}}$$

$$= 63.84$$

The principal/present value is \$63.84.

10. We arrange the data on the following diagram.



By finding the present values of both debts, we can find an equivalent payment that can be paid today to settle both debts.

$$\text{The present value of the \$400 debt} = \frac{400}{\left(1+\frac{0.12}{4}\right)^1} = 388.35$$

$$\text{The value at time 6 of the \$500 debt} = \frac{500}{\left(1+\frac{0.1}{12}\right)^6} = 475.71$$

$$\text{The present value of the \$500 debt} = \frac{475.71}{\left(1+\frac{0.12}{4}\right)^2} = 448.40$$

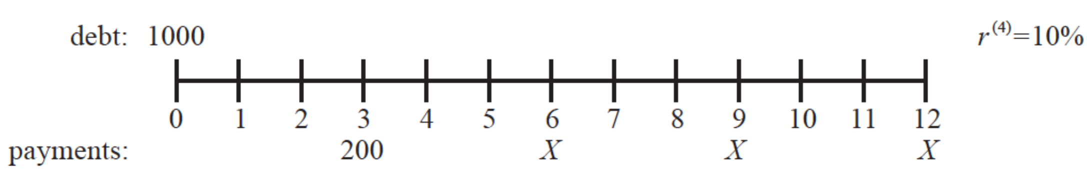
$$\text{Hence the present value of both debts} = 388.35 + 448.40 = 836.75$$

The solution may also be expressed as

$$\text{Present value of both debts} = \frac{400}{\left(1+\frac{0.12}{4}\right)^1} + \frac{500}{\left(1+\frac{0.1}{12}\right)^6\left(1+\frac{0.12}{4}\right)^2} = 836.75$$

A payment of \$836.75 today will repay the debts.

11. We arrange the data on the following diagram.



There are multiple ways to calculate X.

One way is to make sure the present value of the debt is equal to the present values of the four payments at time 0:

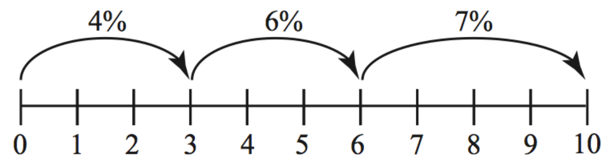
$$
\begin{aligned}
\text{present value of payments} &= \text{present value of debt} \\
\frac{200}{\left(1+\frac{0.1}{4}\right)^1} + \frac{X}{\left(1+\frac{0.1}{4}\right)^2} + \frac{X}{\left(1+\frac{0.1}{4}\right)^3} + \frac{X}{\left(1+\frac{0.1}{4}\right)^4} &= 1000 \\
195.12 + 0.951814396X + 0.928599411X + 0.905950645X &= 1000 \\
195.12 + (0.951814396 + 0.928599411 + 0.905950645)X &= 1000 \\
195.12 + 2.786364452X &= 1000 \\
195.12 + 2.786364452X - 195.12 &= 1000 - 195.12 \\
2.786364452X &= 804.88 \\
\frac{2.786364452X}{2.786364452} &= \frac{804.88}{2.786364452} \\
X &= 288.86
\end{aligned}
$$

Another way is to make sure the accumulated value of the debt is equal to the accumulated values of the four payments at time 12:

$$
\begin{aligned}
\text{accumulated value of payments} &= \text{accumulated value of debt} \\
200\left(1+\frac{0.1}{4}\right)^3 + X\left(1+\frac{0.1}{4}\right)^2 + X\left(1+\frac{0.1}{4}\right)^1 + X &= 1000\left(1+\frac{0.1}{4}\right)^4 \\
215.38 + 1.050625X + 1.025X + X &= 1103.81 \\
215.38 + (1.050625 + 1.025 + 1)X &= 1103.81 \\
215.38 + 3.075625X &= 1103.81 \\
215.38 + 3.075625X - 215.38 &= 1103.81 - 215.38 \\
3.075625X &= 888.43 \\
\frac{3.075625X}{3.075625} &= \frac{888.43}{3.075625} \\
X &= 288.86
\end{aligned}
$$

So the second, third , and fourth payments are equal to \$288.86.

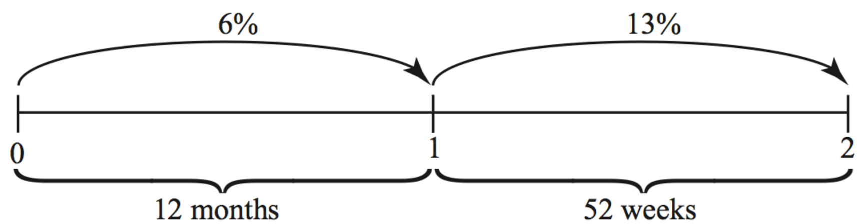
12. (a)



$$\text{Accumulated value after 10 years} = 1000(1+0.04)^3(1+0.06)^3(1+0.07)^4 = 1756.11$$

The accumulated value is \$1756.11.

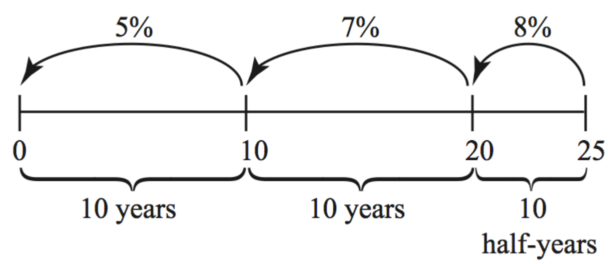
(b)



$$\text{Accumulated value after 2 years} = 500\left(1+\frac{0.06}{12}\right)^{12}\left(1+\frac{0.13}{52}\right)^{52} = 604.44$$

The accumulated value is \$604.44.

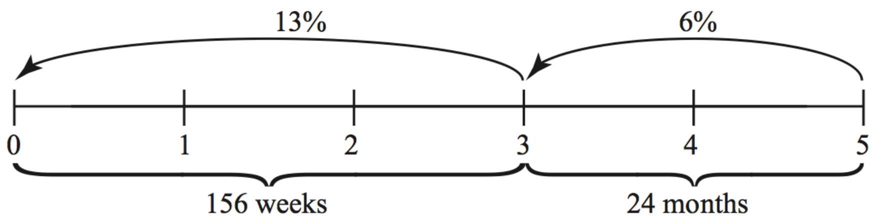
13. (a)



$$\text{Present value} = \frac{1000}{(1+0.05)^{10}(1+0.07)^{10}\left(1+\frac{0.08}{2}\right)^{10}} = 210.83$$

The principal/present value is \$210.83.

(b)



$$\text{Present value} = \frac{1000}{\left(1+\frac{0.13}{52}\right)^{156}\left(1+\frac{0.06}{12}\right)^{24}}$$
$$= 600.97$$

The principal/present value is \$600.97.