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Enriching Mathematics and
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Grade 7 & 8 Math Circles

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Logic Puzzles Solutions

Examples

I.

Sudoku Solution - Easy

9	4	1	8	7	3	6	5	2
7	6	8	4	5	2	1	9	3
3	2	5	1	6	9	7	4	8
6	9	3	5	1	7	8	2	4
8	5	7	6	2	4	9	3	1
2	1	4	9	3	8	5	6	7
4	3	6	7	9	1	2	8	5
1	8	9	2	4	5	3	7	6
5	7	2	3	8	6	4	1	9

www.sudoku-puzzles.net

II.

1	3	4	5	2
3	2	5	1	4
5	4	3	2	1
2	5	1	4	3
4	1	2	3	5

III.

2	x	x	x	x
4	x		3	
x	x		0	
4	5	4		1
x	x	x	x	

IV. A four digit number looks like □□□□, where each box is an integer from 0 to 9, inclusive.

Because the first clue states that the first digit is one-third of the second we know that the second digit must be either 3, 6, or 9.

	□	□	□	□
Option 1	1	3		
Option 2	2	6		
Option 3	3	9		

The second clue states that the third digit is the sum of the first and second digits.

	□	□	□	□
Option 1	1	3	4	
Option 2	2	6	8	
Option 3	3	9	12	

Option 3 is no longer possible because 12 is not an integer between 0 and 9, inclusive.

The third and final clue states that the fourth digit is three times the second digit.

	□	□	□	□
Option 1	1	3	4	9
Option 2	2	6	8	18

Option 2 is no longer possible because 18 is not an integer between 0 and 9, inclusive.

Thus the four digit number must be 1349.

V. Open the door labelled "Unicorns and Radioactive Dragons". Behind this door must be either only unicorns or only radioactive dragons because all the doors are labelled incorrectly.

If you see only unicorns behind the third door then you know that the first door leads to only radioactive dragons and the second door leads to both.

If you see only radioactive dragons behind the third door then you know that the second door leads to only unicorns and the first door leads to both.

See the lecture video for a more thorough solution.

VI. Observe that Dean's statements don't **conclusively** narrow down the suspects. In fact, it cannot be that his first statement "It was Sam" is true, since then "It wasn't Ellen" would be false, in which case it would mean that it was both Sam and Ellen. This is a contradiction. So his first statement "It was Sam" must be false, meaning all we know is that it was not Sam or Ellen.

Consider Jo's statements:

1. It was Dean
2. It was Bobby.

One of these statements must be true. So the culprit is either Dean, or Bobby.

Let's **assume** it was Bobby.

Bobby's statements are:

1. It wasn't Dean
2. It wasn't Sam.

We have two cases: one of these statements has to be false. If the first is false, then it means the culprit was actually Dean! But this means both Dean and Bobby were the thieves! This is a contradiction.

If the second is false, then it means the culprit was Sam! But then Sam and Bobby are both thieves. Again, a contradiction.

Both cases are impossible. Then this must mean that the entire situation, that is, the assumption that Bobby did it, was impossible. So we have to throw it out.

Finishing up, we said it was either Dean or Bobby. But we showed it was impossible to be Bobby. So it must be Dean. Dean stole the pie.

VII. The prime factors of 36 are 2, 2, 3, 3; in other words, $2 \times 2 \times 3 \times 3 = 36$.

By finding the prime factorization of 36, we can construct the following table more easily. The following are all possible ages for the three children:

Age 1	Age 2	Age 3	Sum
1	1	36	38
1	2	18	21
1	3	12	16
1	4	9	14
1	6	6	13
2	2	9	13
2	3	6	11
3	3	4	10

Because the census taker said that knowing the total (from the number on the gate) did not help, we know that knowing the sum of the ages does not give a definitive answer; thus, there must be more than one solution with the same total.

Only two sets of possible ages add up to the same totals:

- $1 \times 6 \times 6 = 36$
- $2 \times 2 \times 9 = 36$

In the first case, there is no 'eldest child' - two children are aged six. Therefore, when told that one child is the eldest, the census-taker concludes that the correct solution is the second case.

VIII.

Abigail	February	Monday
Brenda	December	Wednesday
Mary	June	Sunday
Paula	March	Friday
Tara	July	Saturday

For more information, see http://www.puzzlersparadise.com/article1021.html

Problem Set

1. (a)

Sudoku Solution - Medium

2	5	3	6	9	8	7	1	4
6	9	1	2	7	4	8	3	5
7	8	4	5	3	1	9	2	6
5	1	7	4	2	3	6	9	8
4	3	6	7	8	9	1	5	2
9	2	8	1	6	5	4	7	3
1	4	2	8	5	7	3	6	9
8	6	9	3	1	2	5	4	7
3	7	5	9	4	6	2	8	1

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(b)

Sudoku Solution - Hard

4	5	9	2	6	8	1	3	7
6	7	1	3	5	9	4	8	2
2	3	8	7	1	4	5	9	6
1	9	2	4	7	5	3	6	8
3	8	5	9	2	6	7	4	1
7	4	6	1	8	3	9	2	5
5	6	4	8	3	7	2	1	9
9	2	7	6	4	1	8	5	3
8	1	3	5	9	2	6	7	4

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2.

6	9	6	8	8		9	8	7	5		9	6	0	0
2	3	5	8	7		2	0	3	9		2	2	2	0
6	2	2	6	4		6	3	1	1		6	8	2	4
9	0	6	1				4	0	1	5	7	6	9	5
			1	5	4	2		8	8	8		9	3	8
0	3	7	2	2	4	3	1		8	9	9			
5	1	8		1	8	3	1	7		6	9	7	8	6
9	9	4	9		6	7	1	2	1		8	3	8	0
6	6	3	9	8		2	9	8	8	5		6	9	4
			4	7	2		9	0	1	5	6	2	0	3
4	6	1		0	5	1		8	5	3	7			
6	9	7	8	9	0	4	6				0	3	8	5
7	7	0	1		3	3	3	1		1	9	6	8	8
7	5	1	6		8	7	7	8		2	6	1	3	0
8	3	2	8		6	2	9	7		7	8	9	4	8

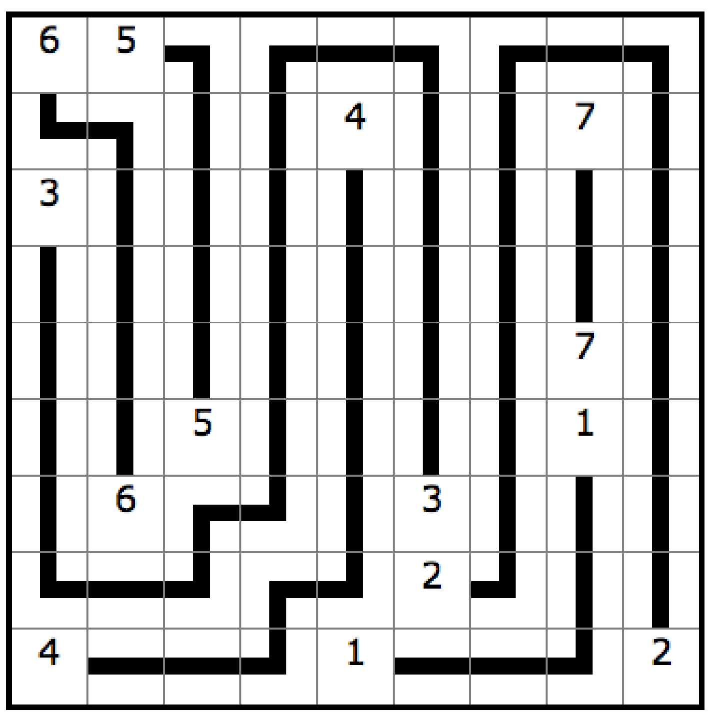
3. (a)

1	3	4	6
2	13	5	7
14	12	8	9
16	15	11	10

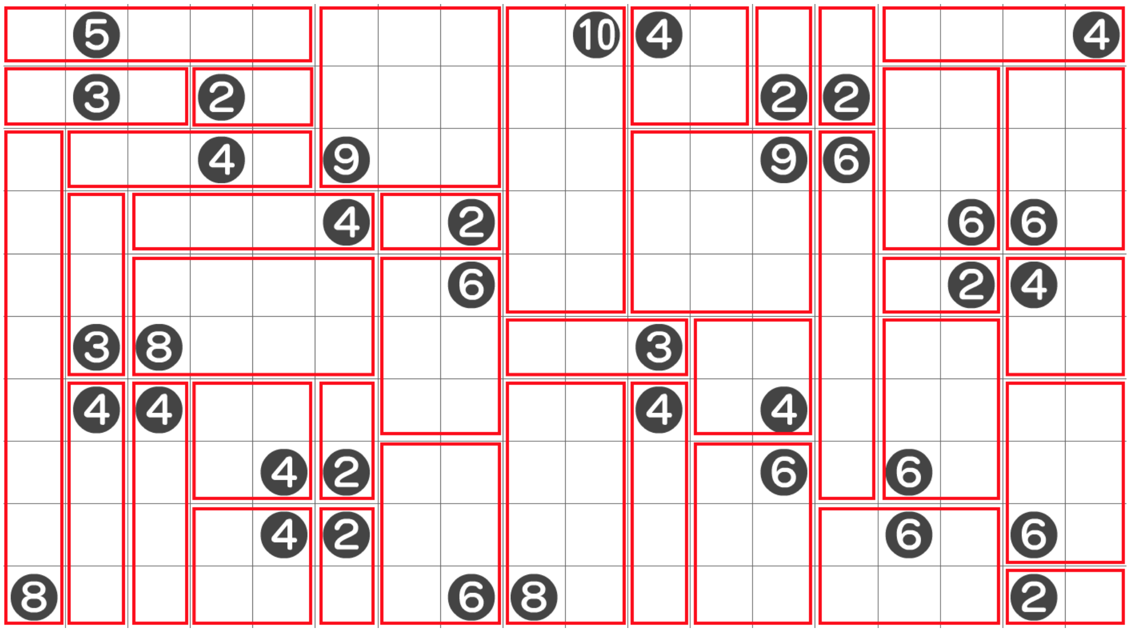
(b)

17	19	21	22	24	25
18	16	20	23	28	26
15	13	12	29	30	27
14	11	6	5	32	31
10	7	36	33	4	3
9	8	35	34	1	2

4.



5.



6.

7	1	6	2	4	3	5
2	6	3	1	5	7	4
3	4	5	6	2	1	7
1	2	7	4	6	5	3
4	7	1	5	3	6	2
5	3	4	7	1	2	6
6	5	2	3	7	4	1

7.

1		(x)	2			1	(x)
(x)		2		(x)	3		
2		(x)		2	(x)	(x)	2
	(x)	3		2			(x)
1			(x)		1	(x)	2
1			2		2	2	
1	(x)	2	(x)		(x)		
	1			2		1	

8. Mr. Blue could only be wearing white or red and we know that there is already someone else wearing the white shirt so Mr. Blue could only be wearing the red shirt.

   Mr. White could have only been wearing a blue or a red shirt, and red is already taken, so Mr. White is wearing a blue shirt.

   Mr. Red now has to be wearing a white shirt.

9. We consider the possibilities and see where it leads us.

   The Gold Chest is either true, or false.

   Gold Chest - False

   If the Gold Chest is false, then its statement *"Exactly one of these two inscriptions is true"* is false. This means that either **both** statements are true, or **both** statements are false.

   Note that **both** statements can't be true, since we assumed the Gold Chest was false. If both statements are false, then the Silver Chest's statement *"This chest contains the python"* is also false. That means the Silver Chest contains the treasure.

   Gold Chest - True

   If the Gold Chest is true, then its statement *"Exactly one of these two inscriptions is true"* is true. But this means that the Silver Chest statement has to be false (since only one is allowed to be true). This means that the statement *"This chest contains the python"* is false. That means the Silver Chest contains the treasure.

   In all possible cases, the Silver chest contains the treasure. Since we considered all the cases, it must mean that the Silver chest actually contains the treasure.

10.

Harold	Derek	Eddie
Bret	Fred	Greg
Alex	John	Chris

11. (a) Consider the possibilities.

    If the first door is true, then the second is false. Since the first door is true, Harry is behind the first door, and Voldemort the second. But then this means the second door is in fact **true** (Harry is behind one of the doors, and Voldemort is behind the other)! This is a **contradiction**. So this cannot be the case.

    So it must be that the first door is false, and the second door true. The second door tells us that one room has to have Harry, and the other room has to have Voldemort.

    If the first room had Harry, then the second room has Voldemort. But this means the first door's sign is also true, which is not the case.

    Therefore, as Sherlock Holmes said, *"When you eliminate the impossible, whatever remains must be the answer"*. The only remaining possibility is that the first room has Voldemort, and the second room has Harry.

    (b) If both are false, then the first door's sign is false. This means that Harry is behind **none** of the doors, which means Voldemort must be behind both. The second door's sign is also false, so Voldemort is **not** behind it. This is a contradiction, since Voldemort has to be behind both if the first sign is false.

    So both signs are true. The second door tells us that the first room has Voldemort. The first door tells us that Harry is behind at least one, but the only room left is the second room, since Voldemort is in the first. Therefore Voldemort is behind the first, Harry behind the second.

12. There are 3 people: one is the grandfather, one is the father, one is the son. The grandfather and the father are both fathers; the father and the son are both sons.

13.

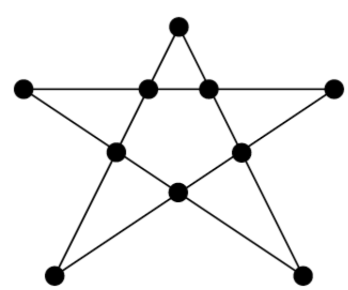
Action	3 L Bucket	5 L Bucket
Fill up 3L	3	-
Pour 3L into 5L	-	3
Fill 3L	3	3
Pour 3L into 5L	1	5
Empty 5L	1	-
Pour 1L into 5L	-	1
Fill 3L	3	1
Pour 3L	-	4

14. Let's call the 7 minute hourglass A, and the 11 minute hourglass B. Turn A and B over simultaneously. Once A runs out, 7 minutes have passed - at this moment, flip A over again. At this point, B has $11 - 7 = 4$ minutes left to go.

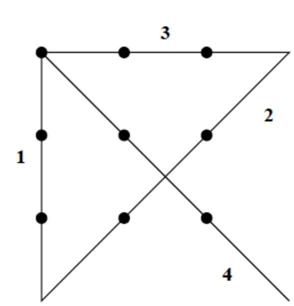
    Wait until B finishes draining these remaining 4 minutes. At this point 11 minutes have passed. Immediately flip over A again (even though it has not fully drained).

    Since A was flipped 4 minutes ago, it will have drained 4 minutes worth of sand, i.e. the bottom half has 4 minutes of sand. By flipping it over again, 4 minutes of sand are now in the top. Just wait for those 4 minutes to finish draining. At this point, $11 + 4 = 15$ minutes will have passed.

15. Arrange the trees in a star shape.



16. See diagram below.



17. Suppose that there were one blue hat. The person with that hat would see two white hats, and since the king specified that there is at least one blue hat, that wise man would immediately know the color of his hat. However, the other two would see one blue and one white hat and would not be able to immediately infer any information from their observations. Therefore, this scenario would violate the king's specification that the contest would be fair to each. So there must be at least two blue hats.

    Suppose then that there were two blue hats. Each wise man with a blue hat would see one blue and one white hat. Since they have already realized that there must be at least two blue hats, they would then immediately know that each were wearing a blue hat. However, the man with the white hat would see two blue hats and would not be able to immediately infer any information from his observations. This scenario, then, would also violate the specification that the contest would be fair to each. So there must be three blue hats.

    Since there must be three blue hats, the first man to figure that out will stand up and say blue.

18. Since at least one inscription is true, and at least one is false, we either have

    - Two chests true, one chest false
    - Two chests false, one chest true

    Consider the sign on the Gold Chest. If the Gold Chest is true, then the Bronze Chest is true. If the Gold Chest is false, then the Bronze Chest is false, but then so is the silver chest! This cannot be, since at least one chest must be true.

    Therefore the Gold Chest is true, and so is the Bronze Chest. This means the Silver Chest has to be false, which means the treasure is not in the Silver Chest. But by the true Gold Chest sign, the treasure is not in the Gold Chest either. This means the treasure is in the Bronze Chest.

19. This problem is best approached by first considering a smaller example.

    Consider a hallway with 10 light switches and 10 passes. Let

Pass	1	2	3	4	5	6	7	8	9	10
1	On	On	On	On	On	On	On	On	On	On
2	On	Off	On	Off	On	Off	On	Off	On	Off
3	On	Off	Off	Off	On	On	On	Off	Off	Off
4	On	Off	Off	On	On	On	On	On	Off	Off
5	On	Off	Off	On	Off	On	On	On	Off	On
6	On	Off	Off	On	Off	Off	On	On	Off	On
7	On	Off	Off	On	Off	Off	Off	On	Off	On
8	On	Off	Off	On	Off	Off	Off	Off	Off	On
9	On	Off	Off	On	Off	Off	Off	Off	On	On
10	On	Off	Off	On	Off	Off	Off	Off	On	Off

Note that the only switches still on are 1, 4, 9. These are special - they are **perfect squares**. Why does this happen?

All switches start in the off position. A switch is flipped (on or off) on every pass which is a factor of the number. For example, the 8th switch is flipped on the 1st, 2nd, 4th, and 8th pass since 8 has factors of 1, 2, 4, 8.

At the end of all the passes, those switches who are flipped an **even** number of times stay off (think about this). Those flipped an odd number of times will be on. This translates to saying, those switches whose numbers have an **even** number of factors will stay on, and those with an **odd** number will stay on.

Which numbers have an odd number of factors? Note that for any number, factors always come in pairs. However, for perfect squares, one of the pairs is made up of the same number. This means perfect squares have a bunch of pairs of different factors (an even amount), plus one factor. This means they have an odd number of factors. For example, 36 has factor pairs (1, 36), (2, 18), (3, 12), (4, 9), (6, 6), which gives nine factors 1, 2, 3, 4, 6, 9, 12, 18, 36.

This reasoning then tell us immediately that in the original problem, the perfect squares less than or equal to 50 will remain on. Note that 1 is a perfect square. That means switches 1, 4, 9, 16, 25, 36, 49 are the only lights still on after the 50th pass.

20. Barry Hathaway hit Nathaniel, drank apple juice and ate peanut butter cookies.
    Curtis Jones hit Harry, drank water and ate chocolate cookies.
    Harry West hit Randy, drank milk and ate sugar cookies.
    Nathaniel Horn hit Curtis, drank orange juice and ate chocolate chip cookies.
    Randy Mann hit Barry, drank iced tea and ate oatmeal raisin cookies.