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Enriching Mathematics and Computer Science for 50 years

Grade 7 & 8 Math Circles
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Algebra Solutions

Exercises I

(a) $$\begin{aligned} 15 \div 3 - 1 &= 5 - 1 \\ &= 4 \end{aligned}$$

(b) $$\begin{aligned} 7 \times 3 + 4 &= 21 + 4 \\ &= 25 \end{aligned}$$

(c) $$\begin{aligned} 48 \div 4 + 4 \times 5 &= 12 + 20 \\ &= 32 \end{aligned}$$

(d) $$\begin{aligned} 48 \div (4 + 4) \times 5 &= 48 \div 8 \times 5 \\ &= 6 \times 5 \\ &= 30 \end{aligned}$$

(e) $$\begin{aligned} 11 - 6^2 \div 12 + (4 + 5 \times 2) \div 7 &= 11 - 6^2 \div 12 + (4 + 10) \div 7 \\ &= 11 - 6^2 \div 12 + 14 \div 7 \\ &= 11 - 36 \div 12 + 14 \div 7 \\ &= 11 - 3 + 2 \\ &= 8 + 2 \\ &= 10 \end{aligned}$$

Exercises II

(a) $$\begin{aligned} 8 \times 27 &= 8 \times (20 + 7) \\ &= 8 \times 20 + 8 \times 7 \\ &= 160 + 56 \\ &= 216 \end{aligned}$$

(b) $14 \times 7 = (10 + 4) \times 7$
$$= 10 \times 7 + 4 \times 7$$
$$= 70 + 28$$
$$= 98$$

(c) $14 \times 27 = (10 + 4) \times (20 + 7)$
$$= 10 \times 20 + 10 \times 7 + 4 \times 20 + 4 \times 7$$
$$= 200 + 70 + 80 + 28$$
$$= 378$$

	10	4
20	200	80
7	70	28

$200 + 80 + 70 + 28 = 378$

(d) $89 \times 76 = (80 + 9) \times (70 + 6)$
$$= 80 \times 70 + 80 \times 6 + 9 \times 70 + 9 \times 6$$
$$= 5600 + 480 + 630 + 54$$
$$= 6764$$

	80	9
70	5600	630
6	480	54

$5600 + 630 + 480 + 54 = 6764$

(e) $(a + b + c) \times (d + e + f) = ad + ae + af + bd + be + bf + cd + ce + cf$

	a	b	c
d	ad	bd	cd
e	ae	be	ce
f	af	bf	cf

$$ad + bd + cd + ae + be + ce + af + bf + cf$$

Exercises III

(a) $35 = 5 \times 7$

(b) $36 = 3 \times 12 = 3 \times 3 \times 4 = 3 \times 3 \times 2 \times 2$

(c) $144 = 12 \times 12 = 3 \times 4 \times 3 \times 4 = 3 \times 2 \times 2 \times 3 \times 2 \times 2$

Factor a common divisor out of the following sums.

(d) $14 + 63 + 35$
All three numbers are multiples of 7, so $14 + 63 + 35 = 7(2 + 9 + 5)$

(e) $6 + 54 + 12 + 48 + 18 + 42 + 24 + 36 + 30$

All nine numbers are multiples of 6, so

$6 + 54 + 12 + 48 + 18 + 42 + 24 + 36 + 30 = 6(1 + 9 + 2 + 8 + 3 + 7 + 4 + 6 + 5)$

You could also factor out 2 or 3 instead of 6. This is because $2 \times 3 = 6$.

Exercises IV

(a)
$$\begin{aligned} x + 3 &= 2 \\ x + 3 - 3 &= 2 - 3 \\ x &= -1 \end{aligned}$$

(b)
$$\begin{aligned} 2 - x &= -3 \\ 2 - x + x &= -3 + x \\ 2 + 3 &= -3 + x + 3 \\ 5 &= x \end{aligned}$$

(c)
$$\begin{aligned} -3x + 7 &= -8 \\ -3x + 7 - 7 &= -8 - 7 \\ \frac{-3x}{-3} &= \frac{-15}{-3} \\ x &= 5 \end{aligned}$$

(d)
$$\begin{aligned} \frac{x}{10} + 5 &= 7 \\ \frac{x}{10} + 5 - 5 &= 7 - 5 \\ \frac{x}{10} \times 10 &= 2 \times 10 \\ x &= 20 \end{aligned}$$

(e)
$$\begin{aligned} 2 - x + 8 - y &= -x + y + 3 - 2 - x \\ 10 - x - y &= -2x + y + 1 \\ 10 - x - y + 2x &= -2x + y + 1 + 2x \\ 10 + x - y - 10 + y &= y + 1 - 10 + y \\ x &= 2y - 9 \end{aligned}$$

(f)
$$\frac{10}{x} + 1 = \frac{9}{4}$$
$$\frac{10}{x} + 1 - 1 = \frac{9}{4} - 1$$
$$\frac{10}{x} \times x = \frac{5}{4} \times x$$
$$10 \times \frac{4}{5} = \frac{5}{4} \times x \times \frac{4}{5}$$
$$8 = x$$

Exercises V

(a)

$$x - 2y = 6 \quad (1)$$
$$3x + y = 25 \quad (2)$$

First, solve (1) for x.

$$x - 2y + 2y = 6 + 2y$$
$$x = 6 + 2y$$

Now replace x in (2) with $6 + 2y$ and solve for y.

$$3(6 + 2y) + y = 25$$
$$18 + 6y + y = 25$$
$$18 + 7y = 25$$
$$18 + 7y - 18 = 25 - 18$$
$$\frac{7y}{7} = \frac{7}{7}$$
$$y = 1$$

Substitute $y = 1$ into (1) and solve for x.

$$x - 2(1) = 6$$
$$x - 2 + 2 = 6 + 2$$
$$x = 8$$

$x = 8$ and $y = 1$ satisfy both (1) and (2), so $x = 8$ and $y = 1$ are the solutions to the system.

(b)

$$2x + 3y = 17 \quad (1)$$
$$-x - y = -4 \quad (2)$$

First, solve (2) for y.

$$\begin{aligned} -x - y + y &= -4 + y \\ -x + 4 &= -4 + y + 4 \\ -x + 4 &= y \end{aligned}$$

Now replace y in (1) with $-x + 4$ and solve for x.

$$\begin{aligned} 2x + 3(-x + 4) &= 17 \\ 2x - 3x + 12 &= 17 \\ -x + 12 &= 17 \\ -x + 12 + x &= 17 + x \\ 12 - 17 &= 17 + x - 17 \\ -5 &= x \end{aligned}$$

Substitute $x = -5$ into (2) and solve for y.

$$\begin{aligned} -(-5) - y &= -4 \\ 5 - y + y &= -4 + y \\ 5 + 4 &= -4 + y + 4 \\ 9 &= y \end{aligned}$$

$x = -5$ and $y = 9$ satisfy both (1) and (2), so $x = -5$ and $y = 9$ are the solutions to the system.

Problem Set

1. (a) $$\begin{aligned}(2+3)\times 5^2-(15-20\div 5)^2 &= 5\times 5^2-(15-4)^2\\ &= 5\times 5^2-11^2\\ &= 5\times 25-121\\ &= 125-121\\ &= 4\end{aligned}$$

   (b) $$\begin{aligned}((-2+2^3)\times 4-(10\div 2)^2)^2 &= ((-2+8)\times 4-(10\div 2)^2)^2\\ &= (6\times 4-5^2)^2\\ &= (6\times 4-25)^2\\ &= (24-25)^2\\ &= (-1)^2\\ &= 1\end{aligned}$$

   (c) $$\begin{aligned}(2^3\times 3^2-9^2)\div 3 &= (8\times 9-81)\div 3\\ &= (72-81)\div 3\\ &= -9\div 3\\ &= -3\end{aligned}$$

   (d) $$\begin{aligned}2(30-(10-(4+12\div 4))^3)-6 &= 2(30-(10-(4+3))^3)-6\\ &= 2(30-(10-7)^3)-6\\ &= 2(30-3^3)-6\\ &= 2(30-27)-6\\ &= 2\times 3-6\\ &= 6-6\\ &= 0\end{aligned}$$

2. The sum of 3 and 7 is $3+7=10$.
   The product of 4 and 8 is $4\times 8=32$.
   But $32-10=22$.
   Therefore the sum of 3 and 7 is smaller than the product of 4 and 8 by 22.

3. Since a, b, c, and d are equal and their sum is 16, then each must have value 4.

$$\begin{aligned} a+b+c+d &= 16 \\ a+a+a+a &= 16 \\ \frac{4a}{4} &= \frac{16}{4} \\ a &= 4 \end{aligned}$$

Therefore the value of $a \times b \times c \times d$ is $4 \times 4 \times 4 \times 4 = 256$.

4. Your numbers and target:

$$\boxed{75}\ \boxed{2}\ \boxed{5}\ \boxed{6}\ \boxed{1}\ \boxed{4}$$

$$\boxed{273}$$

One way:

$$\begin{aligned} 75 \times 4 - (5 \times 6 - 2 - 1) &= 75 \times 4 - (30 - 2 - 1) \\ &= 75 \times 4 - 27 \\ &= 300 - 27 \\ &= 273 \end{aligned}$$

Another way:

$$\begin{aligned} (75 - 6) \times 4 - 2 - 1 &= 69 \times 4 - 2 - 1 \\ &= 276 - 2 - 1 \\ &= 273 \end{aligned}$$

Note: These two options may not be the only ways to reach the target.

5. Use the definition, with $p = 7$ and $q = 5$.

$$\begin{aligned} 7 \odot 5 &= 7^2 + 3 \times 7 \times 5 - 2 \times 5 + 1 \\ &= 49 + 3 \times 7 \times 5 - 2 \times 5 + 1 \\ &= 49 + 105 - 10 + 1 \\ &= 145 \end{aligned}$$

6. (a) $3 \times 57 = 3 \times (50 + 7)$
$$= 3 \times 50 + 3 \times 7$$
$$= 150 + 21$$
$$= 171$$

(b) $5 \times 371 = 5 \times (300 + 70 + 1)$
$$= 5 \times 300 + 5 \times 70 + 5 \times 1$$
$$= 1500 + 350 + 5$$
$$= 1855$$

(c) $\frac{1}{2} \times 1256 = \frac{1}{2} \times (1000 + 200 + 50 + 6)$
$$= \frac{1000}{2} + \frac{200}{2} + \frac{50}{2} + \frac{6}{2}$$
$$= 500 + 100 + 25 + 3$$
$$= 628$$

(d) $23 \times 32 = (20 + 3) \times (30 + 2)$
$$= 20 \times 30 + 20 \times 2 + 3 \times 30 + 3 \times 2$$
$$= 600 + 40 + 90 + 6$$
$$= 736$$

(e) $17 \times 142 = (10 + 7) \times (100 + 40 + 2)$
$$= 10 \times 100 + 10 \times 40 + 10 \times 2 + 7 \times 100 + 7 \times 40 + 7 \times 2$$
$$= 1000 + 400 + 20 + 700 + 280 + 14$$
$$= 2414$$

7. (a)
$$2x + 3 = -7$$
$$2x + 3 - 3 = -7 - 3$$
$$\frac{2x}{2} = \frac{-10}{2}$$
$$x = -5$$

(b)
$$
\begin{aligned}
x + 4 - 2x &= 3x + 20 \\
-x + 4 &= 3x + 20 \\
-x + 4 - 3x &= 3x + 20 - 3x \\
-4x + 4 - 4 &= 20 - 4 \\
\frac{-4x}{-4} &= \frac{16}{-4} \\
x &= -4
\end{aligned}
$$

(c)
$$
\begin{aligned}
\frac{x}{4} + 3 &= \frac{15}{4} \\
\frac{x}{4} + 3 - 3 &= \frac{15}{4} - 3 \\
\frac{x}{4} \times 4 &= \frac{3}{4} \times 4 \\
x &= 3
\end{aligned}
$$

8. Since $2 \times 4 \times 6 \times x = 2 + 4 + 6 + x$, just solve for x.

$$
\begin{aligned}
2 \times 4 \times 6 \times x &= 2 + 4 + 6 + x \\
48x &= 12 + x \\
48x - x &= 12 + x - x \\
\frac{47x}{47} &= \frac{12}{47} \\
x &= \frac{12}{47}
\end{aligned}
$$

9. Clark scored $\frac{1}{4} \times 36$ or 9 points in his first game. In his second game, he scored $\frac{1}{6} \times 36$ or 6 points. In his third game, he scored $\frac{2}{9} \times 36$ or 8 points. In the fourth game, he scored $36 - (9 + 6 + 8)$ or 13 points.

10. We must first solve the system

$$
\begin{aligned}
x - 2y &= -12 && (1) \\
\frac{x}{2} + y &= 8 && (2)
\end{aligned}
$$

First, solve (2) for y.

$$
\begin{aligned}
\frac{x}{2} + y - \frac{x}{2} &= 8 - \frac{x}{2} \\
y &= 8 - \frac{x}{2}
\end{aligned}
$$

Now replace y in (1) with $8 - \frac{x}{2}$ and solve for x.

$$\begin{aligned} x - 2(8 - \frac{x}{2}) &= -12 \\ x - 16 + x &= -12 \\ 2x - 16 &= -12 \\ 2x - 16 + 16 &= -12 + 16 \\ \frac{2x}{2} &= \frac{4}{2} \\ x &= 2 \end{aligned}$$

Substitute $x = 2$ into (2) and solve for y.

$$\begin{aligned} \frac{(2)}{2} + y &= 8 \\ 1 + y - 1 &= 8 - 1 \\ y &= 7 \end{aligned}$$

$x = 2$ and $y = 7$ satisfy both (1) and (2), so $x = 2$ and $y = 7$ are the solutions to the system. Substituting these values into $\frac{y}{3x+8}$ gives:

$$\begin{aligned} \frac{(7)}{3(2)+8} &= \frac{7}{6+8} \\ &= \frac{7}{14} \\ &= \frac{1}{2}. \end{aligned}$$

11. (a) $$\begin{aligned} (a+b)^2 &= (a+b) \times (a+b) \\ &= aa + ab + ba + bb \\ &= a^2 + 2ab + b^2 \end{aligned}$$

 (b) $$\begin{aligned} (a-b)^2 &= (a-b) \times (a-b) \\ &= aa - ab - ba + bb \\ &= a^2 - 2ab + b^2 \end{aligned}$$

 (c) $$\begin{aligned} (a+b) \times (b-a) &= ab - aa + bb - ba \\ &= b^2 - a^2 \end{aligned}$$

(d) $(a+b) \times (a-b) = aa - ab + ba - bb$
$$= a^2 - b^2$$

12. Since no three tins contain the same item, and only one tin contains cocoa, there are two tins of coffee. The tin containing cocoa has a weight that is half the weight of two other tins combined. The only sums of weights that need to be considered are those having 0 as the units digit. The possible sums are:

$$950 + 750 = 1700$$
$$950 + 550 = 1500$$
$$750 + 550 = 1300$$
$$475 + 325 = 800.$$

Of these, the only sum that is double one of the given weights is 1500.

Thus the tin of cocoa is B.