Exercises I
(a) $15 \div 3 - 1 = 5 - 1$
   = 4

(b) $7 \times 3 + 4 = 21 + 4$
   = 25

(c) $48 \div 4 + 4 \times 5 = 12 + 20$
   = 32

(d) $48 \div (4 + 4) \times 5 = 48 \div 8 \times 5$
   = $6 \times 5$
   = 30

(e) $11 - 6^2 \div 12 + (4 + 5 \times 2) \div 7 = 11 - 6^2 \div 12 + (4 + 10) \div 7$
   = $11 - 6^2 \div 12 + 14 \div 7$
   = $11 - 36 \div 12 + 14 \div 7$
   = $11 - 3 + 2$
   = 8 + 2
   = 10

Exercises II
(a) $8 \times 27 = 8 \times (20 + 7)$
   = $8 \times 20 + 8 \times 7$
   = 160 + 56
   = 216
(b) \(14 \times 7 = (10 + 4) \times 7\)
\[= 10 \times 7 + 4 \times 7\]
\[= 70 + 28\]
\[= 98\]

(c) \(14 \times 27 = (10 + 4) \times (20 + 7)\)
\[= 10 \times 20 + 10 \times 7 + 4 \times 20 + 4 \times 7\]
\[= 200 + 70 + 80 + 28\]
\[= 378\]

(d) \(89 \times 76 = (80 + 9) \times (70 + 6)\)
\[= 80 \times 70 + 80 \times 6 + 9 \times 70 + 9 \times 6\]
\[= 5600 + 480 + 630 + 54\]
\[= 6764\]

(e) \((a + b + c) \times (d + e + f) = ad + ae + af + bd + be + bf + cd + ce + cf\)

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\[ad + bd + cd + ae + be + ce + af + bf + cf\]

Exercises III

(a) \(35 = 5 \times 7\)

(b) \(36 = 3 \times 12 = 3 \times 3 \times 4\)

(c) \(144 = 12 \times 12 = 3 \times 4 \times 3 \times 4 = 3 \times 2 \times 2 \times 3 \times 2 \times 2\)

Factor a common divisor out of the following sums.

(d) \(14 + 63 + 35\)

All three numbers are multiples of 7, so \(14 + 63 + 35 = 7(2 + 9 + 5)\)
(e) $6 + 54 + 12 + 48 + 18 + 42 + 24 + 36 + 30$
   All nine numbers are multiples of 6, so
   $6 + 54 + 12 + 48 + 18 + 42 + 24 + 36 + 30 = 6(1 + 9 + 2 + 8 + 3 + 7 + 4 + 6 + 5)$
   You could also factor out 2 or 3 instead of 6. This is because $2 \times 3 = 6$.

Exercises IV

(a) $x + 3 = 2$
   $x + 3 - 3 = 2 - 3$
   $x = -1$

(b) $2 - x = -3$
   $2 - x + x = -3 + x$
   $2 + 3 = -3 + x + 3$
   $5 = x$

(c) $-3x + 7 = -8$
   $-3x + 7 - 7 = -8 - 7$
   $-3x = -15$
   $\frac{-3x}{-3} = \frac{-15}{-3}$
   $x = 5$

(d) $\frac{x}{10} + 5 = 7$
   $\frac{x}{10} + 5 - 5 = 7 - 5$
   $\frac{x}{10} \times 10 = 2 \times 10$
   $x = 20$

(e) $2 - x + 8 - y = -x + y + 3 - 2 - x$
   $10 - x - y = -2x + y + 1$
   $10 - x - y + 2x = -2x + y + 1 + 2x$
   $10 + x - y - 10 + y = y + 1 - 10 + y$
   $x = 2y - 9$
\begin{align*}
(f) & \quad \frac{10}{x} + 1 = \frac{9}{4} \\
& \quad \frac{10}{x} + 1 - 1 = \frac{9}{4} - 1 \\
& \quad \frac{10}{x} \times x = \frac{5}{4} \times x \\
& \quad 10 \times \frac{4}{5} = \frac{5}{4} \times x \times \frac{4}{5} \\
& \quad 8 = x
\end{align*}

Exercises V

(a)

\begin{align*}
& x - 2y = 6 \quad (1) \\
& 3x + y = 25 \quad (2)
\end{align*}

First, solve (1) for \(x\).

\begin{align*}
& x - 2y + 2y = 6 + 2y \\
& x = 6 + 2y
\end{align*}

Now replace \(x\) in (2) with \(6 + 2y\) and solve for \(y\).

\begin{align*}
& 3(6 + 2y) + y = 25 \\
& 18 + 6y + y = 25 \\
& 18 + 7y = 25 \\
& 18 + 7y - 18 = 25 - 18 \\
& 7y = 7 \\
& \frac{7y}{7} = \frac{7}{7} \\
& y = 1
\end{align*}

Substitute \(y = 1\) into (1) and solve for \(x\).

\begin{align*}
& x - 2(1) = 6 \\
& x - 2 + 2 = 6 + 2 \\
& x = 8
\end{align*}

\(x = 8\) and \(y = 1\) satisfy both (1) and (2), so \(x = 8\) and \(y = 1\) are the solutions to the system.
(b)

\[ 2x + 3y = 17 \]  \hspace{1cm} (1)

\[ -x - y = -4 \]  \hspace{1cm} (2)

First, solve (2) for \( y \).

\[ -x - y + y = -4 + y \]
\[ -x + 4 = -4 + y + 4 \]
\[ -x + 4 = y \]

Now replace \( y \) in (1) with \(-x + 4\) and solve for \( x \).

\[ 2x + 3(-x + 4) = 17 \]
\[ 2x - 3x + 12 = 17 \]
\[ -x + 12 = 17 \]
\[ -x + 12 + x = 17 + x \]
\[ 12 - 17 = 17 + x - 17 \]
\[ -5 = x \]

Substitute \( x = -5 \) into (2) and solve for \( y \).

\[ -(-5) - y = -4 \]
\[ 5 - y + y = -4 + y \]
\[ 5 + 4 = -4 + y + 4 \]
\[ 9 = y \]

\( x = -5 \) and \( y = 9 \) satisfy both (1) and (2), so \( x = -5 \) and \( y = 9 \) are the solutions to the system.
Problem Set

1. (a) \((2 + 3) \times 5^2 - (15 - 20 \div 5)^2 = 5 \times 5^2 - (15 - 4)^2\)
   
   \[= 5 \times 5^2 - 11^2\]
   
   \[= 5 \times 25 - 121\]
   
   \[= 125 - 121\]
   
   \[= 4\]

(b) \((-2 + 2^3) \times 4 - (10 \div 2)^2 = ((-2 + 8) \times 4 - (10 \div 2)^2)^2\)

\[= (6 \times 4 - 5^2)^2\]

\[= (6 \times 4 - 25)^2\]

\[= (24 - 25)^2\]

\[= (-1)^2\]

\[= 1\]

(c) \((2^3 \times 3^2 - 9^2) \div 3 = (8 \times 9 - 81) \div 3\)

\[= (72 - 81) \div 3\]

\[= -9 \div 3\]

\[= -3\]

(d) \(2(30 - (10 - (4 + 12 \div 4))^3) - 6 = 2(30 - (10 - (4 + 3))^3) - 6\)

\[= 2(30 - (10 - 7)^3) - 6\]

\[= 2(30 - 3^3) - 6\]

\[= 2(30 - 27) - 6\]

\[= 2 \times 3 - 6\]

\[= 6 - 6\]

\[= 0\]

2. The sum of 3 and 7 is \(3 + 7 = 10\).
   
The product of 4 and 8 is \(4 \times 8 = 32\).
   
   But \(32 - 10 = 22\).
   
   Therefore the sum of 3 and 7 is smaller than the product of 4 and 8 by 22.
3. Since $a$, $b$, $c$, and $d$ are equal and their sum is 16, then each must have value 4.

\[
\begin{align*}
    a + b + c + d &= 16 \\
    a + a + a + a &= 16 \\
    4a &= 16 \\
    a &= 4
\end{align*}
\]

Therefore the value of $a \times b \times c \times d$ is $4 \times 4 \times 4 \times 4 = 256$.

4. Your numbers and target:

\[
\begin{array}{cccccc}
    7 & 5 & 2 & 6 & 1 & 4 \\
\end{array}
\]

\[
273
\]

One way:

\[
\begin{align*}
    75 \times 4 - (5 \times 6 - 2 - 1) &= 75 \times 4 - (30 - 2 - 1) \\
    &= 75 \times 4 - 27 \\
    &= 300 - 27 \\
    &= 273
\end{align*}
\]

Another way:

\[
\begin{align*}
    (75 - 6) \times 4 - 2 - 1 &= 69 \times 4 - 2 - 1 \\
    &= 276 - 2 - 1 \\
    &= 273
\end{align*}
\]

Note: These two options may not be the only ways to reach the target.

5. Use the definition, with $p = 7$ and $q = 5$.

\[
\begin{align*}
    7 \odot 5 &= 7^2 + 3 \times 7 \times 5 - 2 \times 5 + 1 \\
    &= 49 + 3 \times 7 \times 5 - 2 \times 5 + 1 \\
    &= 49 + 105 - 10 + 1 \\
    &= 145
\end{align*}
\]
6. (a) \[3 \times 57 = 3 \times (50 + 7)\]
    \[= 3 \times 50 + 3 \times 7\]
    \[= 150 + 21\]
    \[= 171\]

(b) \[5 \times 371 = 5 \times (300 + 70 + 1)\]
    \[= 5 \times 300 + 5 \times 70 + 5 \times 1\]
    \[= 1500 + 350 + 5\]
    \[= 1855\]

(c) \[\frac{1}{2} \times 1256 = \frac{1}{2} \times (1000 + 200 + 50 + 6)\]
    \[= \frac{1000}{2} + \frac{200}{2} + \frac{50}{2} + \frac{6}{2}\]
    \[= 500 + 100 + 25 + 3\]
    \[= 628\]

(d) \[23 \times 32 = (20 + 3) \times (30 + 2)\]
    \[= 20 \times 30 + 20 \times 2 + 3 \times 30 + 3 \times 2\]
    \[= 600 + 40 + 90 + 6\]
    \[= 736\]

(e) \[17 \times 142 = (10 + 7) \times (100 + 40 + 2)\]
    \[= 10 \times 100 + 10 \times 40 + 10 \times 2 + 7 \times 100 + 7 \times 40 + 7 \times 2\]
    \[= 1000 + 400 + 20 + 700 + 280 + 14\]
    \[= 2414\]

7. (a) \[2x + 3 = -7\]
    \[2x + 3 - 3 = -7 - 3\]
    \[2x = -10\]
    \[\frac{2x}{2} = \frac{-10}{2}\]
    \[x = -5\]
(b) \[ x + 4 - 2x = 3x + 20 \]
\[ -x + 4 = 3x + 20 \]
\[ -x + 4 - 3x = 3x + 20 - 3x \]
\[ -4x + 4 - 4 = 20 - 4 \]
\[ \frac{-4x}{-4} = \frac{16}{-4} \]
\[ x = -4 \]

(c) \[ \frac{x}{4} + 3 = \frac{15}{4} \]
\[ \frac{x}{4} + 3 - 3 = \frac{15}{4} - 3 \]
\[ \frac{x}{4} \times 4 = \frac{3}{4} \times 4 \]
\[ x = 3 \]

8. Since \[ 2 \times 4 \times 6 \times x = 2 + 4 + 6 + x \], just solve for \( x \).

\[ 2 \times 4 \times 6 \times x = 2 + 4 + 6 + x \]
\[ 48x = 12 + x \]
\[ 48x - x = 12 + x - x \]
\[ \frac{47x}{47} = \frac{12}{47} \]
\[ x = \frac{12}{47} \]

9. Clark scored \( \frac{1}{4} \times 36 \) or 9 points in his first game. In his second game, he scored \( \frac{1}{6} \times 36 \) or 6 points. In his third game, he scored \( \frac{2}{9} \times 36 \) or 8 points. In the fourth game, he scored \( 36 - (9 + 6 + 8) \) or 13 points.

10. We must first solve the system

\[ x - 2y = -12 \] \hspace{1cm} (1)
\[ \frac{x}{2} + y = 8 \] \hspace{1cm} (2)

First, solve (2) for \( y \).

\[ \frac{x}{2} + y - \frac{x}{2} = 8 - \frac{x}{2} \]
\[ y = 8 - \frac{x}{2} \]
Now replace $y$ in (1) with $8 - \frac{x}{2}$ and solve for $x$.

\[
x - 2\left(8 - \frac{x}{2}\right) = -12
\]
\[
x - 16 + x = -12
\]
\[
2x - 16 = -12
\]
\[
2x - 16 + 16 = -12 + 16
\]
\[
\frac{2x}{2} = \frac{4}{2}
\]
\[
x = 2
\]

Substitute $x = 2$ into (2) and solve for $y$.

\[
\frac{(2)}{2} + y = 8
\]
\[
1 + y - 1 = 8 - 1
\]
\[
y = 7
\]

$x = 2$ and $y = 7$ satisfy both (1) and (2), so $x = 2$ and $y = 7$ are the solutions to the system. Substituting these values into $\frac{y}{3x + 8}$ gives:

\[
\frac{(7)}{3(2) + 8} = \frac{7}{6 + 8}
\]
\[
= \frac{7}{14}
\]
\[
= \frac{1}{2}
\]

11. (a) $(a + b)^2 = (a + b) \times (a + b)$
\[
= aa + ab + ba + bb
\]
\[
= a^2 + 2ab + b^2
\]

(b) $(a - b)^2 = (a - b) \times (a - b)$
\[
= aa - ab - ba + bb
\]
\[
= a^2 - 2ab + b^2
\]

(c) $(a + b) \times (b - a) = ab - aa + bb - ba$
\[
= b^2 - a^2$
(d) \((a + b) \times (a - b) = aa - ab + ba - bb = a^2 - b^2\)

12. Since no three tins contain the same item, and only one tin contains cocoa, there are two tins of coffee. The tin containing cocoa has a weight that is half the weight of two other tins combined. The only sums of weights that need to be considered are those having 0 as the units digit. The possible sums are:

\[
\begin{align*}
950 + 750 &= 1700 \\
950 + 550 &= 1500 \\
750 + 550 &= 1300 \\
475 + 325 &= 800.
\end{align*}
\]

Of these, the only sum that is double one of the given weights is 1500.

Thus the tin of cocoa is B.