Intermediate Math Circles
March 19, 2014
Cryptography I

Jen Nelson
jen.nelson@uwaterloo.ca

Centre for Education in Mathematics and Computing
Faculty of Mathematics
University of Waterloo
Waterloo, Canada

cemc.uwaterloo.ca

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Intermediate Math Circles - Night at a Glance

1. What is Cryptography?
2. A bit on Modular Arithmetic
3. Let's do some encryption and decryption!
4. Let's break a cipher!

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- **Caesar's Shift Cipher** was used by Julius Caesar in 50 B.C. to communicate with his military

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In the 20th century, cryptography played a significant role in many global conflicts (eg. Enigma machine and Bletchley Park in WWII).

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The need for secret communication is even larger than it was in the recent past.

Credit card, debit card and web transactions, as well as privacy concerns for the electronic storage of health, citizenship and other records, have raised the need for secure communications and secure storage dramatically.

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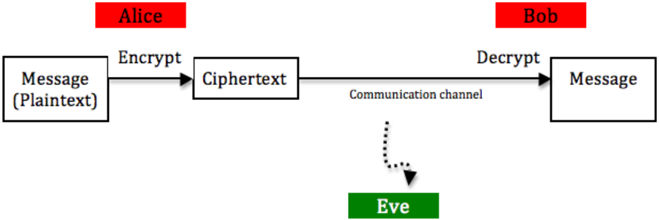

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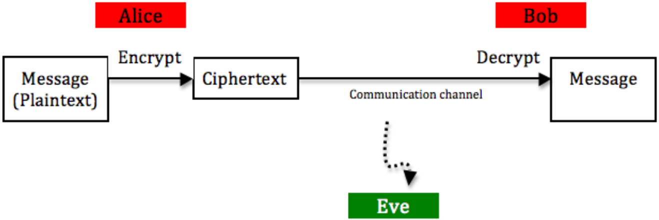

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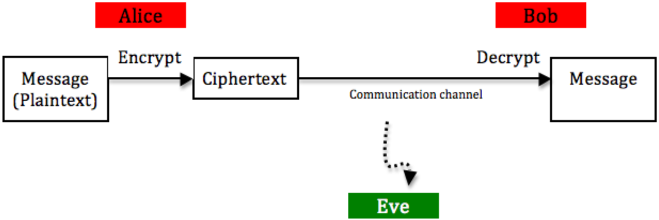

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Alice wants to use a simple algorithm, so that Bob can "decrypt" the transmitted message using some special key that only he has, but so that it is hard for Eve to "break the code" without knowing the key.

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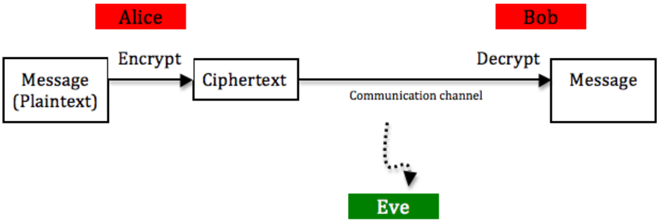

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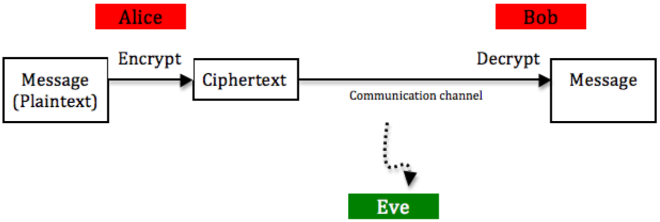

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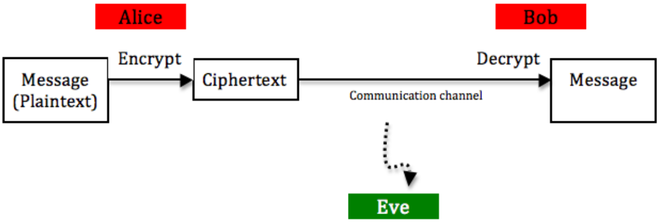

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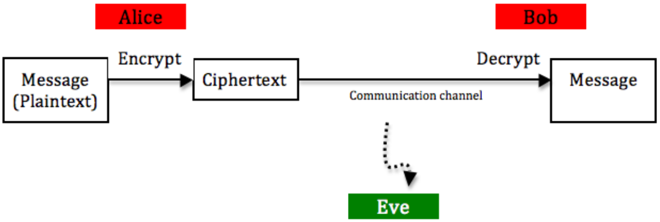

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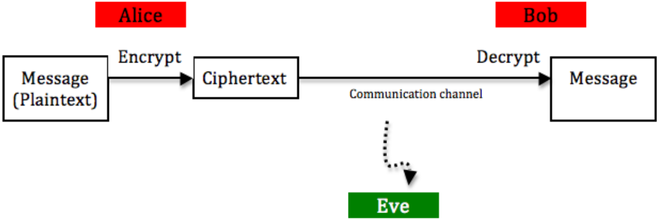

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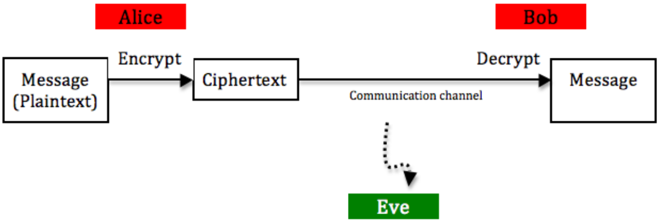

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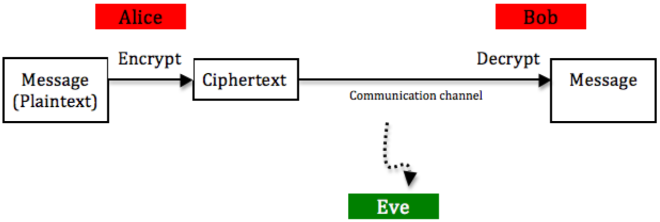

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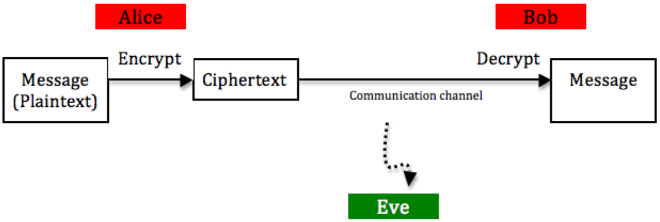

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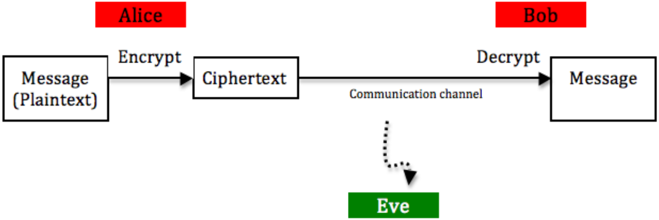

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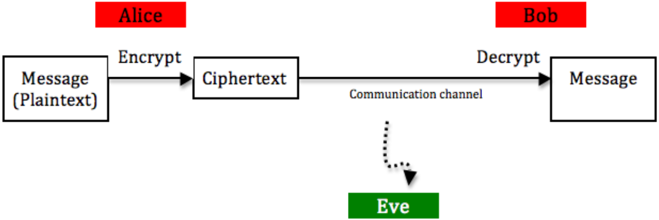

The security of the code should depend on keeping the **key** a secret, not keeping the encryption algorithm a secret.

Alice should be able to code her message with a well-known method and still be reasonably confident that the message cannot be decoded by anyone other than Bob, since he is the only person with the key.

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We will be interested in the smallest integer $b \geq 0$ such that
$a - b = km$, where k is some integer.

Example 1

Reduce

a) $52 \mod 8$

b) $41 \mod 5$

c) $84 \mod 4$

d) $-17 \mod 4$

e) $145672 \mod 13$

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(i) $a + b \equiv A + B \pmod{m}$

(ii) $a - b \equiv A - B \pmod{m}$

Reduce (in two different ways)

a) $17 + 21 \mod 6$

b) $83 - 21 \mod 3$

c) $21 - 83 \mod 11$

Exercise Set 1

1. Reduce 237288 modulo 5
2. Reduce $192 + 118$ modulo 5
3. Reduce $192 - 118$ modulo 5
4. Reduce $118 - 192$ modulo 5
5. Today is a Wednesday. What day of the week will it be
   a) 100 days from now?
   b) 365 days from now?
   c) 1000 days from now?
6. Emily celebrated her 13th birthday on Wednesday, February 19th, 2014. On what day of the week was she born? (Don't forget about the leap years in 2004, 2008 and 2012!)

Exercise Set 1: Answers

Answers to Exercise Set 1:

1. $237288 \equiv 3 \mod 5$
2. $192 + 118 \equiv 0 \mod 5$
3. $192 - 118 \equiv 4 \mod 5$
4. $118 - 192 \equiv 1 \mod 5$
5. a) Friday
   b) Thursday
   c) Tuesday
6. Monday

Substitution Ciphers: A couple of examples

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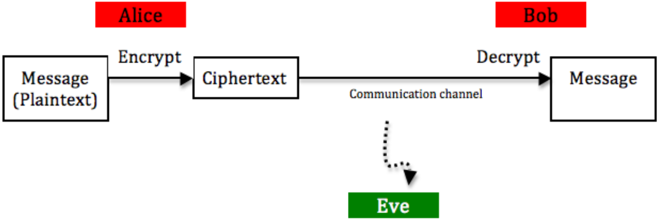

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Decryption Algorithm:

The Caesar Shift Cipher:

Assign the numbers 0 to 25 to the letters A to Z (so A is 0, B is 1 and so on, Z is 25).
Think of the alphabet mod 26.
Pick a random number to be your **key**, call it k.

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How does Bob decrypt the ciphertext into plaintext?

Example 3

a) Using a Caesar Shift Cipher and secret key 11, encrypt the message "I WANT COOKIES".

b) Using a Caesar Shift Cipher and secret key 11, decrypt the message "NSPNV ESP NFAMZLCO".

A	B	C	D	E	F	G	H	I	J	K	L	M

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Exercise Set 2

1. Encode the message "MODULAR ARITHMETIC" using a Caesar Shift cipher with secret key $k = 7$.
2. Decode the message "SLA AOLT LHA JHRL" using a Caesar Shift cipher with secret key $k = 7$.
3. Encode the message "ALL SQUARES ARE RECTANGLES" using a Caesar Shift cipher with secret key $k = 14$.
4. Decode the message "PIH BCH OZZ FSQHOBUZSG OFS GEIOFSG" using a a Caesar Shift cipher with secret key $k = 14$.
5. Find a partner to work with. Think of a secret message to send to them and encode it using a Caesar Shift cipher with a secret key of your choice. Give your partner the coded message and shift number. Decode your partner's message to you.

Answers to Exercise Set 2:

1. TVKBSHY HYPAOTLAPJ
2. LET THEM EAT CAKE
3. OZZ GEIOFSG OFS FSQHOBUZSG
4. BUT NOT ALL RECTANGLES ARE SQUARES

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The table above acts as the **key** for this cipher.

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How many possible keys are there? $26! \approx 4 \times 10^{26}$
Breaking the cipher by trying all keys is no longer feasible, even for a computer!

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X	E	N	M	H	T	D	S	I	L	F	R	A

Example 4:

a) Using the Substitution Cipher above, encrypt the message "I WANT COOKIES".

b) Using the Substitution Cipher above, decrypt the message "BX DWK CEEUBK QOH".

The Substitution Cipher is better than the Caesar Shift Cipher, but unfortunately, it can also be easily broken.

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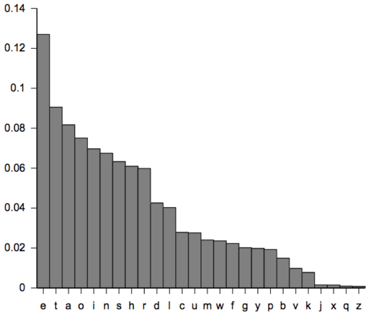

Order of relatively frequency:

1. E
2. T, A, O, I, N, S, H, R
3. D, L
4. C, U, M, W, F, G, Y P, B
5. V, K, J, X, Q, Z

Exercise Set 3 Using known letter frequencies in the English alphabet, try to break the code below.
The message was encrypted with a substitution cipher.

```
IFYYOL PYZZR AXRGVK QWBZ IQL IFWK FB ZWEV PXB QB MFL
ZWV ZJ BIV JVM BIFB KQK. IVYRQZWV DYFWDVYL IFK LQRGUO
YZUUVK ZHVY ZW BIV DYZXWK FWK WVHQUUVL IFKWB RZHVK FB
FUU. GVYIFGL PYZZRL UQTV IZYLVL EZXUK BVUU MIVW OZX
MVYV FJYFQK BIZXDIB IFYYO BIVYV MFL F SXFHVY QW
WVHQUUVL HZQEV BIFB LFQK ZWUO BZZ EUVFYUO BIFB IV
MFWBVK BZ TVVG IQL JVVB ZW BIV DYZXWK.
RFKFR IZZEI BIVW LIZMVK BIVR IZM BZ RZXWB BIVQY
PYZZRL MQBIZXB LUQKQWD ZJJ BIV VWK FWK MFUTVK XG FWK
KZMW BIV YZML EZYYVEBQWD BIVQY DYQGL.
```

Exercise Set 3

Here are the frequencies of each letter in this example:

A	B	C	D	E	F	G	H	I	J	K	L	M
1	34	0	8	7	28	7	6	32	6	23	19	12

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	6	4	21	11	1	3	18	45	26	11	27	38

Next Week

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Next week we will begin our build up to another encryption scheme: RSA Encryption.