Intermediate Math Circles
February 19, 2014
Contest Preparation III

The Pascal, Cayley and Fermat Contest are all written, Thursday, February 20, 2014.

The two warm-up problems that we will look at have been taken from past Pascal and Cayley contests.

Warm-Up #1: 2006 Pascal #24

A bag contains eight yellow marbles, seven red marbles, and five black marbles. Without looking in the bag, Igor removes N marbles all at once. If he is to be sure that, no matter which choice of N marbles he removes, there are at least four marbles of one colour and at least three marbles of another colour left in the bag, what is the maximum possible value of N?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Since the solution can be found online, it will not be reprinted here. However, as a quick check, the correct answer is (B) 7.

Note: Since we are looking for the maximum value of N, one strategy would be to start with the largest answer to see if it works. Then progress through the answers until the correct answer is determined.

Warm-Up #2: 2007 Cayley #24

The number 8 is the sum and product of the numbers in the collection of four positive integers $\{1, 1, 2, 4\}$, since $1 + 1 + 2 + 4 = 8$ and $1 \times 1 \times 2 \times 4 = 8$. The number 2007 can be made up from a collection of n positive integers that multiply to 2007 and add to 2007. What is the smallest value of n with $n > 1$?

(A) 1171 (B) 1337 (C) 1551 (D) 1777 (E) 1781

Since the solution can be found online, it will not be reprinted here. However, as a quick check, the correct answer is (B) 1337.

For the remainder of the time tonight, we will work on Problem Set #6, the *Australian Mathematics Competition* Warm-Up Paper Intermediate 9, along with three Extra Problems.

Problem Set 6:

1. C 2. A 3. B 4. [-6, 6]

5. $T = 3, U = 2$ and $T = 8, U = 6$

6. 69375 7. C 8. A

9. C 10. E

Australian Mathematics Competition
Warm-Up Paper - Intermediate 9:

1. D 2. E 3. C 4. D 5. B

6. D 7. A 8. A 9. A 10. D

Full solutions will be found online at www.cemc.uwaterloo.ca. Go to web resources and then Math Circles. Go to Math Circles material and then to Winter 2014, Intermediate Feb 19/2014 Solutions.

Extra Problem #1: 1996 Pascal, # 25

There are exactly k perfect squares which are divisors of 1996^{1996}. The sum of the digits in the number k is

(A) 29 (B) 26 (C) 30 (D) 22 (E) 27

I don't think a solution to this question is available online, so you will find a solution in the Math circles material - Intermediate Feb 19/2014 Solutions. The answer is (E) 27.

Extra Problem #2: 2002 AMC 10A, # 25

In trapezoid $ABCD$ with bases AB and CD, we have $AB = 52, BC = 12, CD = 39$, and $DA = 5$. The area of $ABCD$ is

(A) 182 (B) 195 (C) 210 (D) 234 (E) 260

Solution can be found online at http://www.artofproblemsolving.com. Select AoPSWiki from the Resources tab then follow the path: AMC Problems - AMC 10 Problems and Solutions - AMC 10A - Problem 25. The answer is (C) 210.

Extra Problem #3: 1998 Caley, # 25

One way to pack a 100 by 100 square with 10 000 circles, each of diameter 1, is to put them in 100 rows with 100 circles in each row. If the circles are repacked so that the centres of any three tangent circles form an equilateral traingle, what is the maximum number of additional circles that can be packed?

(A) 647 (B) 1442 (C) 1343 (D) 1443 (E) 1344

The answer is (D) 1443.

Good success on whichever contest you write!