Pascal and Cayley Contest Preparation

Problem Set 6

1. The numbers 49, 29, 9, 40, 22, 15, 53, 33, 13, 47 are grouped in pairs so that the sum of each pair is the same. Which number is paired with 15?
   (A) 33 (B) 40 (C) 47 (D) 49 (E) 53

2. When the expression $2005^2 + 2005^0 + 2005^0 + 2005^5$ is evaluated, the final two digits are
   (A) 52 (B) 25 (C) 20 (D) 50 (E) 05

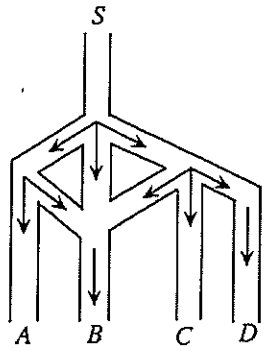
3. The number of positive integers that are less than 500 and that are not divisible by 2 or 3 is
   (A) 168 (B) 167 (C) 166 (D) 165 (E) 83

4. If $4 \leq x \leq 12$ and $6 \leq y \leq 10$, and $S = x - y$, what is the largest possible interval for S?

5. The five-digit number $9T67U$, where T and U are digits, is divisible by 36. Determine all possible values for T and U.

6. Using only odd digits, all possible three-digit numbers are formed. Determine the sum of all such numbers.

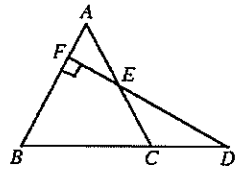
7. Harry the Hamster is put in a maze, and he starts at point S. The paths are such that Harry can move forward only in the direction of the arrows. At any junction, he is equally likely to choose any of the forward paths. What is the probability that Harry ends up at B?



(A) $\frac{2}{3}$ (B) $\frac{13}{18}$ (C) $\frac{11}{18}$ (D) $\frac{1}{3}$ (E) $\frac{1}{4}$

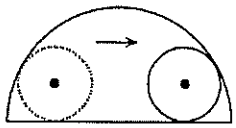
8. If $90! = (90)(89)(88)\cdots(2)(1)$, then the exponent of the highest power of 2 that will divide 90! is
(A) 86 (B) 45 (C) 90 (D) 75 (E) 85

9. In the diagram, $\triangle ABC$ is equilateral, $BC = 2CD$, $AF = 6$, and DEF is perpendicular to AB. What is the area of quadrilateral $FBCE$?



(A) $144\sqrt{3}$ (B) $138\sqrt{3}$ (C) $126\sqrt{3}$ (D) $108\sqrt{3}$ (E) $66\sqrt{3}$

10. A wheel of radius 8 rolls along the diameter of a semicircle of radius 25 until it bumps into this semicircle. What is the length of the portion of the diameter of the semicircle that cannot be touched by the wheel?



(A) 8 (B) 12 (C) 15 (D) 17 (E) 20

AUSTRALIAN MATHEMATICS COMPETITION
WARM-UP PAPER
INTERMEDIATE 9

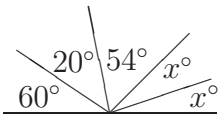
Questions 1 - 4, 3 marks each

1. $(7a + 5b) - (5a - 7b)$ equals

(A) $12a - 12b$ (B) $2a - 2b$ (C) 0 (D) $2a + 12b$ (E) $12a - 2b$

2. In the diagram x equals

(A) 34 (B) 33 (C) 46 (D) 67 (E) 23



3. The value of

$$\frac{\sqrt{20 + x^2}}{\sqrt{20 - x^2}},$$

when $x = 4$, is

(A) $\sqrt{\frac{3}{2}}$ (B) $\frac{9}{4}$ (C) 3 (D) $\frac{9}{2}$ (E) 9

4. What is the highest power of 2 which divides exactly into 1 000 000?

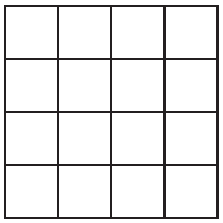
(A) 2^3 (B) 2^4 (C) 2^5 (D) 2^6 (E) 2^8

Questions 5 - 8, 4 marks each

5. A litre of orange fruit juice drink contains 10% orange juice. How many millilitres of orange juice must be added to produce a mixture containing 50% orange juice?

(A) 450 (B) 800 (C) 600 (D) 400 (E) 500

6. Piran has a 4×4 grid of squares on which he is trying to place as many counters as possible.



No more than one counter may be placed on any square and no more than three on any row, column or diagonal. What is the maximum number of counters he can place in this way?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

7. How many positive integers less than 1000 have the sum of their digits equal to 6?

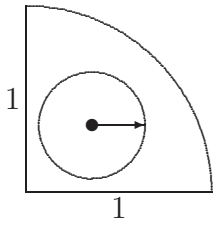
(A) 28 (B) 19 (C) 111 (D) 18 (E) 27

8. Three jolly swagmen, John, Kevin and Robert, are gambling by a billabong. They start with sums of money in the ratio $7 : 6 : 5$ and finish with sums of money in the ratio $6 : 5 : 4$ (in the same order of wealth). One of the swagmen won \$12. How many dollars did he start with?

(A) 420 (B) 1080 (C) 432 (D) 120 (E) 90

Questions 9 - 10, 5 marks each

9. What is the radius of the largest circle that can be drawn inside a quarter circle of radius 1 unit?



(A) $\sqrt{2} - 1$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{1}{\pi}$ (E) $\frac{1}{2\sqrt{2}}$

10. If p and q are positive integers such that

$$\frac{7}{10} < \frac{p}{q} < \frac{11}{15}$$

then the smallest possible value of q is

(A) 25 (B) 60 (C) 30 (D) 7 (E) 6