



Grade 7/8 Math Circles
March 25/26, 2014
Gauss Contest Preparation Solutions

General Information

The Gauss contest is an opportunity for grade 7/8 students to have fun and challenge their mathematical problem solving skills

Date and Registration

Registration Date: April 23, 2014

Test Writing Date: May 14, 2014

Format and Marking Scheme

- 60 minutes
- 25 multiple choice questions
- 150 marks:
 - Part A: 10 questions - 5 marks each
 - Part B: 10 questions - 6 marks each
 - Part C: 5 questions - 8 marks each
 - Unanswered Questions: 2 marks each (for up to 10 questions)

Contest Success Strategies

- **ELIMINATE** - choices that aren't sensible answers, making it easier to guess
- **DRAW** - diagrams representing your scenario to help clear up misconceptions
- **MOVE ON** - from questions you are stuck on to get as many marks as possible
- **FOCUS** - on Part B and Part C questions as Part A shouldn't pose a challenge
- **PRACTICE** - by studying from the contest bank on the CEMC website
- **LEARN** - techniques and short-cuts from past contest solutions

Mock Gauss Contest

Part A - 5 marks each

1. Gauss Grade 8, 2011 (#3)

The number 0.2012 is between

- (A) 0 and $\frac{1}{10}$ (B) $\frac{1}{10}$ and $\frac{1}{5}$ (C) $\frac{1}{5}$ and $\frac{1}{4}$ (D) $\frac{1}{4}$ and $\frac{1}{3}$ (E) $\frac{1}{3}$ and $\frac{1}{2}$

Since the number in question (0.2012) is in decimal form, it is easiest to determine into which of the 5 given ranges it falls by converting the ranges into decimal form also.

Converting, $\frac{1}{10}$ is 0.1, $\frac{1}{5}$ is 0.2, $\frac{1}{4}$ is 0.25, $\frac{1}{3}$ is $0.\bar{3}$, and $\frac{1}{2}$ is 0.5.

Since 0.2012 is greater than 0.2 but less than 0.25, it is between $\frac{1}{5}$ and $\frac{1}{4}$.

ANSWER: C

2. Gauss Grade 8, 2013 (#3)

The smallest number in the set $\{\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{7}{12}\}$ is

- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) $\frac{5}{6}$ (E) $\frac{7}{12}$

Solution 1

To determine the smallest number in the set $\{\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{7}{12}\}$, we express each number with a common denominator of 12.

The set $\{\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{7}{12}\}$ is equivalent to the set $\{\frac{1 \times 6}{2 \times 6}, \frac{2 \times 4}{3 \times 4}, \frac{1 \times 3}{4 \times 3}, \frac{5 \times 2}{6 \times 2}, \frac{7}{12}\}$ or to the set $\{\frac{6}{12}, \frac{8}{12}, \frac{3}{12}, \frac{10}{12}, \frac{7}{12}\}$.

The smallest number in this set is $\frac{3}{12}$, so $\frac{1}{4}$ is the smallest number in the original set.

Solution 2

With the exception of $\frac{1}{4}$, each number in the set is greater than or equal to $\frac{1}{2}$.

We can see this by recognizing that the numerator of each fraction is greater than or equal to one half of its denominator.

Thus, $\frac{1}{4}$ is the only number in the list that is less than $\frac{1}{2}$ and so it must be the smallest number in the set.

ANSWER: C

3. Gauss Grade 7, 2011 (#10)

I bought a new plant for my garden. Anika said it was a red rose, Bill said it was a purple daisy, and Cathy said it was a red dahlia. Each person was correct in stating either the colour or type of plant. What was the plant that I bought?

- (A) purple dahlia (B) purple rose (C) red dahlia
(D) yellow rose (E) red daisy

Anika said that the plant was a *red* rose.

Cathy said that the plant was a *red* dahlia.

If red was not the correct colour of the plant, then Anika was incorrect about the colour and therefore must be correct about the type; in other words, the plant is a rose.

Similarly, if red was not the correct colour of the plant, then Cathy was incorrect about the colour and therefore must be correct about the type; in other words, the plant is a dahlia.

But the plant cannot be both a rose and a dahlia.

Therefore, Cathy and Anika must have been correct about the colour being red.

Bill said that the plant was a purple daisy.

Since we know that the colour of the plant is red, then Bill was incorrect about it being purple.

Therefore, Bill must have been correct about it being a daisy.

Thus, the plant is a red daisy.

ANSWER: E

4. Gauss Grade 7, 2013 (#10)

UVW and XYZ are each 3-digit integers. U, V, W, X, Y and Z are different digits chosen from the integers 1 to 9. What is the largest possible value for $UVW - XYZ$

- (A) 678 (B) 864 (C) 885 (D) 888 (E) 975

To make the difference $UVW - XYZ$ as large as possible, we make UVW as large as possible and XYZ as small as possible.

The hundreds digit of a number contributes more to its value than its tens digit, and its tens digit contributes more to its value than its units digit.

Thus, we construct the largest possible number UVW by choosing 9 (the largest digit) to be its hundreds digit, U , and by choosing 8 (the second largest digit) to be its tens digit, V , and by choosing 7 (the third largest digit) to be the units digit, W .

Similarly, we construct the smallest possible number XYZ by choosing 1 (the smallest allowable digit) to be its hundreds digit, X , and 2 (the second smallest allowable digit) to be its tens digit, Y , and by choosing 3 (the third smallest allowable digit) to be its units digit, Z .

The largest possible difference is $UVW - XYZ$ or $987 - 123$ or 864.

ANSWER: B

Part B - 6 marks each

5. Gauss Grade 7, 2011 (#18)

Three pumpkins are weighed two at a time in all possible ways. The weights of the pairs of pumpkins are 12 kg, 13 kg and 15 kg. How much does the lightest pumpkin weigh?

- (A) 4 kg (B) 5 kg (C) 6 kg (D) 7 kg (E) 8 kg

In kilograms, let the weights of the 3 pumpkins in increasing order be A , B and C . The lightest combined weight, 12 kg, must come from weighing the two lightest pumpkins. That is, $A + B = 12$.

The heaviest combined weight, 15 kg, must come from weighing the two heaviest pumpkins.

That is, $B + C = 15$.

Then the third given weight, 13 kg, is the combined weight of the lightest and heaviest pumpkins.

That is, $A + C = 13$.

Systematically, we may try each of the 5 possible answers.

If the lightest pumpkin weighs 4 kg (answer (A)), then $A + B = 12$ gives $4 + B = 12$, or $B = 8$.

If $B = 8$, then $8 + C = 15$ or $C = 7$.

Since C represents the weight of the heaviest pumpkin, C cannot be less than B and therefore the lightest pumpkin cannot weigh 4 kg.

If the lightest pumpkin weighs 5 kg (answer (B)), then $A + B = 12$ gives $5 + B = 12$, or $B = 7$.

If $B = 7$, then $7 + C = 15$ or $C = 8$.

If $A = 5$ and $C = 8$, then the third and final equation $A + C = 13$ is also true.

Therefore, the weight of the smallest pumpkin must be 5 kg.

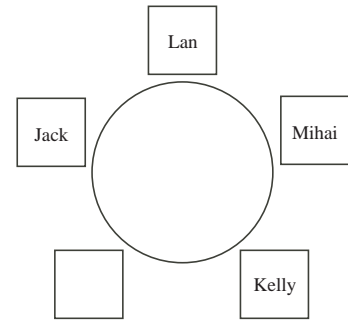
ANSWER: B

6. Gauss Grade 7, 2013 (#13)

Jack, Kelly, Lan, Mihai and Nate are sitting in the 5 chairs around a circular table. Lan and Mihai are sitting beside each other. Jack and Kelly are not sitting beside each other. the 2 people who are seated on either side of Nate are

- (A) Jack and Lan (B) Jack and Kelly (C) Kelly and Mihai
(D) Lan and Mihai (E) Mihai and Jack

Since Lan and Mihai are seated beside each other, while Jack and Kelly are not, the only possible location for the remaining chair (Nate's chair) is between Jack and Kelly, as shown. Therefore, the 2 people who are seated on either side of Nate are Jack and Kelly.



ANSWER: B

7. Gauss Grade 7, 2013 (#19)

A special six-sided die is rolled. The probability of rolling a number that is a multiple of three is $\frac{1}{2}$. The probability of rolling an even number is $\frac{1}{3}$. A possibility for the numbers on the die is

- (A) 1,2,3,5,5,6 (B) 1,2,3,3,5,6 (C) 1,2,3,4,6,6
(D) 1,2,3,3,4,6 (E) 2,3,3,3,5,6

Using the special six-sided die, the probability of rolling a number that is a multiple of three is $\frac{1}{2}$.

Since $\frac{1}{2}$ of 6 is 3, then exactly 3 numbers on the die must be multiples of 3.

Since the probability of rolling an even number is $\frac{1}{3}$ and $\frac{1}{3}$ of 6 is 2, then exactly 2 numbers on the die must be even.

The die in (A) has only 2 numbers that are multiples of 3 (3 and 6), and thus may be eliminated.

The die in (C) has 4 numbers that are even (2, 4, 6, 6), and thus may be eliminated.

The die in (D) has 3 numbers that are even (2, 4, 6), and thus may be eliminated.

The die in (E) has 4 numbers that are multiples of 3 (3, 3, 3, 6), and thus may be eliminated.

The die in (B) has exactly 3 numbers that are multiples of 3 (3, 3, 6), and exactly 2

even numbers (2 and 6), and is therefore the correct answer.

ANSWER: B

8. Gauss Grade 7, 2008 (#18)

Rishi got the following marks on four math tests: 71, 77, 80 and 87. He will write one more math test. Each test is worth the same amount and all marks are between 0 and 100. Which of the following is a possible average for his five math tests?

- (A) 88 (B) 62 (C) 82 (D) 84 (E) 86

The sum of Rishi's marks so far is $71 + 77 + 80 + 87 = 315$.

Since Rishi's mark on his next test is between 0 and 100, the sum of his marks will be between $315 + 0 = 315$ and $315 + 100 = 415$ after his next test.

Since his average equals the sum of his marks divided by the number of marks, then his average will be between $\frac{315}{5} = 63$ and $\frac{415}{5} = 83$.

Of the given choices, the only one in this range is 82.

ANSWER: C

9. Gauss Grade 7, 2012 (#19)

A set of five different positive integers has a mean (average) of 20 and a median of 18. What is the greatest possible integer in the set?

- (A) 60 (B) 26 (C) 46 (D) 12 (E) 61

In an ordered list of five integers, the median is the number in the middle or third position.

Thus if we let the set of integers be a, b, c, d, e , ordered from smallest to largest, then $c = 18$.

Since the average is fixed (at 20), e (the largest number in the set) is largest when a, b and d are as small as possible.

Since the numbers in the set are different positive integers, the smallest that a can be is 1 and the smallest that b can be is 2.

Our set of integers is now 1, 2, 18, d, e .

Again, we want d to be as small as possible, but it must be larger than the median 18. Therefore, $d = 19$.

Since the average of the 5 integers is 20, then the sum of the five integers is $5 \times 20 = 100$. Thus, $1 + 2 + 18 + 19 + e = 100$ or $40 + e = 100$, and so $e = 60$.

The greatest possible integer in the set is 60.

ANSWER: A

10. Gauss Grade 7, 2012 (#20)

Chris lies on Fridays, Saturdays and Sundays, but he tells the truth on all other days. Mark lies on Tuesdays, Wednesdays and Thursdays, but he tells the truth on all other days. On what day of the week would they both say: "Tomorrow I will lie."?

- (A) Monday (B) Thursday (C) Friday (D) Sunday (E) Tuesday

If either Chris or Mark says, "Tomorrow, I will lie." on a day that he tells a lie, then it actually means that tomorrow he will tell the truth (since he is lying).

This can only occur when he lies and then tells the truth on consecutive days.

For Chris, this only happens on Sunday, since he lies on Sunday but tells the truth on Monday.

For Mark, this only happens on Thursday, since he lies on Thursday but tells the truth on Friday.

Similarly, if either Chris or Mark says, "Tomorrow, I will lie." on a day that they tell the truth, then it means that tomorrow they will lie (since they are telling the truth).

This can only occur when they tell the truth and then lie on consecutive days.

For Chris, this only happens on Thursday, since he tells the truth on Thursday but lies on Friday.

For Mark, this only happens on Monday, since he tells the truth on Monday but lies on Tuesday.

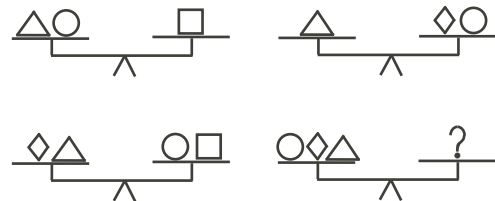
Therefore, the only day of the week that they would both say, "Tomorrow, I will lie.", is Thursday. ANSWER: B

11. Gauss Grade 7, 2012 (#18)

All four scales shown are balanced.

One possible replacement for the ? is

- (A) $\triangle \square$ (B) $\diamond \triangle$ (C) $\circ \square$
 (D) $\square \diamond$ (E) $\triangle \circ$



In turn, we may use each of the three known scales to find a way to balance a circle, a diamond and a triangle.

Since many answers are possible, we must then check our solution to see if it exists among the five answers given.

First consider the scale at the top right.

A diamond and a circle are balanced by a triangle.

If we were to add a triangle to both sides of this scale, then it would remain balanced and the right side of this scale would contain what we are trying to balance, a circle, a diamond and a triangle.

That is, a circle, a diamond and a triangle are balanced by two triangles.

However, two triangles is not one of the five answers given.

Next, consider the scale at the top left.

A triangle and a circle are balanced by a square.

If we were to add a diamond to both sides of this scale, then it would remain balanced and the left side of this scale would contain what we are trying to balance, a circle, a diamond and a triangle.

That is, a circle, a diamond and a triangle are balanced by a square and a diamond.

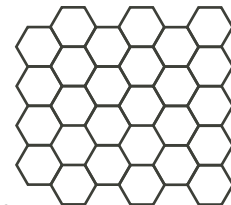
This answer is given as one of the five answers.

In the context of a multiple choice contest, we do not expect that students will verify that the other four answers do not balance a circle, a diamond and a triangle. However, it is worth noting that it can be shown that they do not.

ANSWER: D

12. Gauss Grade 8, 2013 (#19)

Serena colours the hexagons on the tiling shown. If two hexagons share a side, she colours them with different colours. What is the least number of colours that she can use to colour all of the hexagons?



- (A) 4 (B) 6 (C) 7 (D) 2 (E) 3

Since we seek the minimum number of colours that Serena can use to colour the hexagons, we first determine if it is possible for her to use only two colours (using only one colour is not possible). We will use the numbers 1, 2, 3 to represent distinct (different) colours.

We begin by choosing any group of three hexagons in which each pair of hexagons share a side, as shown.

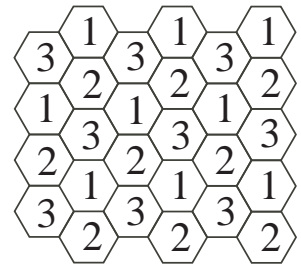


We colour two of the hexagons shown with colours 1 and 2 (since they share a side).

Each of these two coloured hexagons share a side with the third hexagon which therefore can not be coloured 1 or 2.

Thus, the minimum number of colours that Serena can use is at least three.
 Next, we determine if the entire tiling can be coloured using only three colours.

One possible colouring of the tiles that uses only three colours is shown to the right.



While other colourings of the tiles are possible, Serena is able to use only three colours and ensure that no two hexagons that share a side are the same colour.

There are many nice patterns of the colours in this tiling.
 Can you find a different colouring of the tiles that uses only three colours?

ANSWER: D

Part C - 8 marks each

13. Gauss Grade 7, 2012 (#23)

The number N is the product of all positive odd integers from 1 to 99 that do not end in the digit 5. That is, $N = 1 \times 3 \times 7 \times 9 \times 11 \times 13 \times 17 \times 19 \times \dots \times 91 \times 93 \times 97 \times 99$.
 The unit digit of N is

- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

The units digit of any product is given by the units digit of the product of the units digits of the numbers being multiplied.

For example, the units digit of the product 12×53 is given by the product 2×3 , so it is 6.

Thus to determine the units digit of N , we need only consider the product of the units digits of the numbers being multiplied to give N .

The units digits of the numbers in the product N are 1, 3, 7, 9, 1, 3, 7, 9, \dots , and so on. That is, the units digits 1, 3, 7, 9 are repeated in each group of four numbers in the product.

There are ten groups of these four numbers, 1, 3, 7, 9, in the product.

We first determine the units digit of the product $1 \times 3 \times 7 \times 9$.

The units digit of 1×3 is 3.

The units digit of the product 3×7 is 1 (since $3 \times 7 = 21$).

The units digit of 1×9 is 9.

Therefore, the units digit of the product $1 \times 3 \times 7 \times 9$ is 9.

(We could have calculated the product $1 \times 3 \times 7 \times 9 = 189$ to determine the units

digit.)

This digit 9 is the units digits of the product of each group of four successive numbers in N .

Thus, to determine the units digit of N we must determine the units digit of

$$9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9.$$

This product is equal to $81 \times 81 \times 81 \times 81 \times 81$.

Since we are multiplying numbers with units digit 1, then the units digit of the product is 1.

ANSWER: A

14. **Gauss Grade 7, 2011 (#22)**

A pool has a volume of 4000 L. Sheila starts filling the empty pool with water at a rate of 20 L/min. The pool springs a leak after 20 minutes and water leaks out at 2 L/min. Beginning from the time when Sheila starts filling the empty pool, how long does it take until the pool is completely full?

- (A) 3 hours (B) 3 hours 40 minutes (C) 4 hours
(D) 4 hours 20 minutes (E) 3 hours 20 minutes

In the first 20 minutes, Sheila fills the pool at a rate of 20 L/min and thus adds $20 \times 20 = 400$ L of water to the pool.

At this time, the pool needs $4000 - 400 = 3600$ L of water to be full.

After filling for 20 minutes, water begins to leak out of the pool at a rate of 2 L/min. Since water is still entering the pool at a rate of 20 L/min, then the net result is that the pool is filling at a rate of $20 - 2 = 18$ L/min.

Since the pool needs 3600 L of water to be full and is filling at a rate of 18 L/min, then it will take an additional $3600 \div 18 = 200$ minutes before the pool is full of water.

Thus, the total time needed to fill the pool is $20 + 200 = 220$ minutes or 3 hours and 40 minutes.

ANSWER: B

15. Gauss Grade 7, 2013 (#21)

In the addition showed, P and Q each represent single digits, and the sum is $1PP7$. What is $P + Q$?

- (A) 9 (B) 12 (C) 14
 (D) 15 (E) 13

$$\begin{array}{r} 77P \\ 6QP \\ + \quad QQP \\ \hline 1PP7 \end{array}$$

The sum of the units column is $P + P + P = 3P$.

Since P is a single digit, and $3P$ ends in a 7, then the only possibility is $P = 9$.

This gives:

$$\begin{array}{r} 7 \ 7 \ 9 \\ 6 \ Q \ 9 \\ + \ Q \ Q \ 9 \\ \hline 1 \ 9 \ 9 \ 7 \end{array}$$

Then $3P = 3 \times 9 = 27$, and thus 2 is carried to the tens column.

The sum of the tens column becomes $2 + 7 + Q + Q$ or $9 + 2Q$.

Since $9 + 2Q$ ends in a 9 (since $P = 9$), then $2Q$ ends in $9 - 9 = 0$.

Since Q is a single digit, there are two possibilities for Q such that $2Q$ ends in 0.

These are $Q = 0$ and $Q = 5$.

If $Q = 0$, then the sum of the tens column is 9 with no carry to the hundreds column.

In this case, the sum of the hundreds column is $7 + 6 + Q$ or 13 (since $Q = 0$); the units digit of this sum does not match the 9 in the total.

Thus, we conclude that Q cannot equal 0 and thus must equal 5.

Verifying that $Q = 5$, we check the sum of the tens column again.

Since $2 + 7 + 5 + 5 = 19$, then 1 is carried to the hundreds column.

The sum of the hundreds column is $1 + 7 + 6 + 5 = 19$, as required.

Thus, $P + Q = 9 + 5 = 14$ and the completed addition is shown below.

$$\begin{array}{r} 7 \ 7 \ 9 \\ 6 \ 5 \ 9 \\ + \ 5 \ 5 \ 9 \\ \hline 1 \ 9 \ 9 \ 7 \end{array}$$

ANSWER: C

Problem Set

1. Gauss Grade 7, 2008 (#11)

Chris bought two hockey sticks at the same price. He also bought a helmet for \$ 25. if Chris spent \$ 68 in total, how much did one hockey stick cost?

- (A) \$ 9.00 (B) \$ 18.00 (C) \$ 21.50 (D) \$ 43.00 (E) \$ 41.50

Since Chris spent \$68 in total and \$25 on the helmet, then he spent $\$68 - \$25 = \$43$ on the two hockey sticks.

Since the two sticks each cost the same amount, then this cost was $\$43 \div 2 = \21.50 .

ANSWER: C

2. Gauss Grade 7, 2009 (#7)

On a map of Nunavut, a length of 1 centimetre measured on a map represents a real distance of 60 kilometres. What length on the map represents a real distance of 540 kilometres?

- (A) 9 cm (B) 90 cm (C) 0.09 cm (D) 0.11 cm (E) 5.4 cm

Since a real distance of 60 km is represented by 1 cm on the map, then a real distance of 540 km is represented by $\frac{540}{60}$ cm or 9 cm on the map.

ANSWER: C

3. Gauss Grade 7, 2011 (#13)

Five children had dinner. Chris ate more than Max. Brandon ate less than Kayla. Kayla ate less than Max but more than Tanya. Which child ate the second most?

- (A) Brandon (B) Chris (C) Kayla (D) Max (E) Tanya

Since Kayla ate less than Max and Chris ate more than Max, then Kayla ate less than Max who ate less than Chris.

Brandon and Tanya both ate less than Kayla.

Therefore, Max ate the second most.

ANSWER: D

4. Gauss Grade 7, 2011 (#19)

The sum of four numbers is T . Suppose that each of the four numbers is now increased by 1. These four new numbers are added together and then the sum is tripled. What is the value of this final result.

- (A) $3T + 3$ (B) $3T + 4$ (C) $3T + 12$ (D) $T + 12$ (E) $12T$

If each of the four numbers is increased by 1, then the increase in their sum is 4. That is, these four new numbers when added together have a sum that is 4 more than their previous sum T , or $T + 4$.

This new sum $T + 4$ is now tripled.

The result is $3 \times (T + 4) = (T + 4) + (T + 4) + (T + 4)$ or $3T + 12$.

ANSWER: C

5. Gauss Grade 7, 2012 (#11)

The perimeter of a square is 36 cm. The area of the square in cm^2 , is

- (A) 24 (B) 81 (C) 36 (D) 1296 (E) 324

Since the four sides of a square are equal in length and the perimeter is 36 cm, then each side has length $\frac{36}{4} = 9$ cm.

The area of the square is the product of the length and width, which are each equal to 9 cm.

Therefore, the area of the square, in cm^2 , is $9 \times 9 = 81$.

ANSWER: B

6. Gauss Grade 7, 2009 (#13)

The product $60 \times 60 \times 24 \times 7$ equals

- (A) the number of minutes in seven weeks
(B) the number of hours in sixty days
(C) the number of seconds in seven hours
(D) the number of seconds in one week
(E) the number of minutes in twenty-four weeks

There are 60 seconds in a minute, 60 minutes in an hour, 24 hours in a day and 7 days in a week. Therefore, the number of seconds in one week is $60 \times 60 \times 24 \times 7$.

ANSWER: D

7. Gauss Grade 7, 2010 (#16)

In a magic square, all rows, columns, and diagonals have the same sum. The magic square shown uses each of the integers from -6 to +2. What is the value of Y?

- (A) -1 (B) 0 (C) -6
 (D) +2 (E) -2

+1		Y
-4		
-3		-5

First we must find the *magic constant*, that is, the sum of each row, column and diagonal. From column one, we find that the magic constant is $(+1) + (-4) + (-3) = -6$. In the diagonal extending from the top left corner to the bottom right corner, the two existing numbers +1 and -5 have a sum of -4.

Thus, to obtain the magic constant of -6 in this diagonal, -2 must occupy the centre square. In the diagonal extending from the bottom left corner to the top right corner, the two numbers -3 and -2, have a sum of -5.

Thus, to obtain the magic constant of -6 in this diagonal, Y must equal -1.

The completed magic square is shown below.

+1	-6	-1
-4	-2	0
-3	+2	-5

ANSWER: A

8. Gauss Grade 7, 2011 (#16)

A 51 cm rod is built from 5 cm rods and 2 cm rods. All of the 5 cm rods must come first, and are followed by the 2 cm rods. For example the rod could be made from seven 5 cm rods followed by eight 2 cm rods. How many ways are there to build the 51 cm rod?

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Any number of 2 cm rods add to give a rod having an even length.

Since we need an odd length, 51 cm, then we must combine an odd length from the 5 cm rods with the even length from the 2 cm rods to achieve this.

An odd length using 5 cm rods can only be obtained by taking an odd number of them.

All possible combinations are shown in the table below.

Number of 5 cm rods	Length in 5 cm rods	Length in 2 cm rods	Number of 2 cm rods
1	5	$51 - 5 = 46$	$46 \div 2 = 23$
3	15	$51 - 15 = 36$	$36 \div 2 = 18$
5	25	$51 - 25 = 26$	$26 \div 2 = 13$
7	35	$51 - 35 = 16$	$16 \div 2 = 8$
9	45	$51 - 45 = 6$	$6 \div 2 = 3$

Note that attempting to use 11 (or more) 5 cm rods gives more than the 51 cm length required.

Thus, there are exactly 5 possible combinations that add to 51 cm using 5 cm rods first followed by 2 cm rods.

ANSWER: A

9. * Gauss Grade 7, 2010 (#19)

If each of the four numbers 3, 4, 6 and 7 replaces a \square , what is the largest possible sum of the fractions shown?

- (A) $\frac{19}{12}$ (B) $\frac{13}{7}$ (C) $\frac{5}{2}$
(D) $\frac{15}{4}$ (E) $\frac{23}{6}$

$$\frac{\square}{\square} + \frac{\square}{\square}$$

The value of any positive fraction is increased by increasing the numerator and/or decreasing the denominator.

Thus, to obtain the largest possible sum, we choose 6 and 7 as the numerators, and 3 and 4 as the denominators.

We then calculate:

$$\frac{7}{3} + \frac{6}{4} = \frac{28}{12} + \frac{18}{12} = \frac{46}{12} = \frac{23}{6}$$

We recognize that $\frac{23}{6}$ is the largest of the 5 possible answers, and thus is the correct response.

(This means that we do not need to try $\frac{7}{4} + \frac{6}{3}$.)

ANSWER: E

10. * Gauss Grade 7, 2013 (#22)

An *arithmetic sequence* is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, 2, 4, 6, 8 and 1, 4, 7, 10 are arithmetic sequences.

In the grid shown, the numbers in each row must form an arithmetic sequence and the numbers in each column must form an arithmetic sequence. The value of x is

- (A) 37 (B) 28 (C) 36
 (D) 43.75 (E) 46

1			
4			25
7			x
10		36	

We use labels, m and n , in the fourth row of the grid, as shown. Then, 10, m , 36, n are four terms of an arithmetic sequence. Since 10 and 36 are two terms apart in this sequence, and their difference is $36 - 10 = 26$, the constant added to one term to obtain the next term in the fourth row is $\frac{26}{2}$ or 13.

That is, $m = 10 + 13 = 23$, and $n = 36 + 13 = 49$.

(We confirm that the terms 10, 23, 36, 49 do form an arithmetic sequence.)

In the fourth column, 25 and n (which equals 49) are two terms apart in this sequence, and their difference is $49 - 25 = 24$. Thus, the constant added to one term to obtain the next term in the fourth column is $\frac{24}{2}$ or 12.

That is, $x = 25 + 12 = 37$ (or $x = 49 - 12 = 37$).

The completed grid is as shown.

ANSWER: A

1			
4			25
7			x
10	m	36	n

1	5	9	13
4	11	18	25
7	17	27	37
10	23	36	49

11. ** Gauss Grade 7, 2010 (#23)

Molly assigns every letter of the alphabet a different whole number value. She finds the value of a word by *multiplying* the values of its letters together. For example, if D has a value of 10, and I has a value of 8, then the word DID has a value of $10 \times 8 \times 10 = 800$. The table shows the value of some words. What is the value of the word MATH?

Word	Value
TOTE	18
TEAM	168
MOM	49
HOME	70
MATH	?

- (A) 19 (B) 840 (C) 420
 (D) 190 (E) 84

Since $M \times O \times M = 49$, either $M = 7$ and $O = 1$ or $M = 1$ and $O = 49$.

However, since the value of the word TOTE is 18, O cannot have a value of 49 because 18 is not divisible by 49.

Thus, $M = 7$ and $O = 1$.

Since $T \times O \times T \times E = 18$ and $O = 1$, we have $T \times T \times E = 18$.

Therefore, either $T = 3$ and $E = 2$ or $T = 1$ and $E = 18$.

However, $O = 1$, and since every letter has a different value, T cannot be equal to 1.

Thus, $T = 3$ and $E = 2$.

The value of the word TEAM is 168, so $T \times E \times A \times M = 168$, or $3 \times 2 \times A \times 7 = 168$.

Thus, $42 \times A = 168$ or $A = 168 \div 42 = 4$.

The value of the word HOME is 70, so $H \times O \times M \times E = 70$, or $H \times 1 \times 7 \times 2 = 70$.

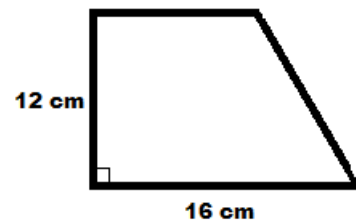
Thus, $14 \times H = 70$ or $H = 70 \div 14 = 5$.

Finally, the value of the word MATH is $M \times A \times T \times H = 7 \times 4 \times 3 \times 5 = 420$.

ANSWER: C

12. ** Gauss Grade 8, 2011 (#23)

The trapezoid shown has a height of length 12 cm, a base of length 16 cm, and an area of 162 cm^2 . The perimeter of the trapezoid is:



- (A) 51 cm (B) 52 cm (C) 49.6 cm
 (D) 50 cm (E) 56 cm

We first label the trapezoid $ABCD$ as shown in the diagram below.

Since AD is the perpendicular height of the trapezoid, then AB and DC are parallel.

The area of the trapezoid is $\frac{AD}{2} \times (AB + DC)$ or $\frac{12}{2} \times (AB + 16)$ or $6 \times (AB + 16)$.

Since the area of the trapezoid is 162, then $6 \times (AB + 16) = 162$ and $AB + 16 = \frac{162}{6}$

or $AB + 16 = 27$, so $AB = 11$.

Construct a perpendicular from B to E on DC .

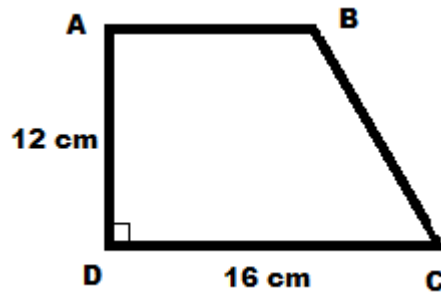
Since AB is parallel to DE and both AD and BE are perpendicular to DE , then $ABED$ is a rectangle.

Thus, $DE = AB = 11$, $BE = AD = 12$, and $EC = DC - DE = 16 - 11 = 5$.

Since $\angle BEC = 90^\circ$, then $\triangle BEC$ is a right-angled triangle.

Thus by the Pythagorean Theorem, $BC^2 = BE^2 + EC^2$ or $BC^2 = 12^2 + 5^2$ or $BC^2 = 169$ so $BC = 13$ (since $BC > 0$).

The perimeter of the trapezoid is $AB + BC + CD + DA = 11 + 13 + 16 + 12 = 52$ cm.



ANSWER: B

13. *** Gauss Grade 7, 2013 (#25)

A box contains a total of 400 tickets that come in five colours: blue, green, red, yellow and orange. The ratio of blue to green to red tickets is $1 : 2 : 4$. The ratio of green to yellow to orange tickets is $1 : 3 : 6$. What is the smallest number of tickets that must be drawn to ensure that at least 50 tickets of one colour have been selected?

- (A) 50 (B) 246 (C) 148 (D) 196 (E) 115

We denote the number of tickets of each of the five colours by the first letter of the colour.

We are given that $b : g : r = 1 : 2 : 4$ and that $g : y : o = 1 : 3 : 6$.

Through multiplication by 2, the ratio $1 : 3 : 6$ is equivalent to the ratio $2 : 6 : 12$.

Thus, $g : y : o = 2 : 6 : 12$.

We chose to scale this ratio by a factor of 2 so that the only colour common to the two given ratios, green, now has the same number in both of these ratios.

That is, $b : g : r = 1 : 2 : 4$ and $g : y : o = 2 : 6 : 12$ and since the term g is 2 in each ratio, then we can combine these to form a single ratio, $b : g : r : y : o = 1 : 2 : 4 : 6 : 12$.

This ratios tells us that for every blue ticket, there are 2 green, 4 red, 6 yellow, and 12 orange tickets.

Thus, if there was only 1 blue ticket, then there would be $1 + 2 + 4 + 6 + 12 = 25$ tickets in total.

However, we are given that the box contains 400 tickets in total.

Therefore, the number of blue tickets in the box is $\frac{400}{25} = 16$.

Through multiplication by 16, the ratio $b : g : r : y : o = 1 : 2 : 4 : 6 : 12$ becomes $b : g : r : y : o = 16 : 32 : 64 : 96 : 192$.

(Note that there are $16 + 32 + 64 + 96 + 192 = 400$ tickets in total.)

Next, we must determine the smallest number of tickets that must be drawn to ensure that at least 50 tickets of one colour have been selected.

It is important to consider that up to 49 tickets of any one colour could be selected without being able to *ensure* that 50 tickets of one colour have been selected.

That is, it is possible that the first 195 tickets selected could include exactly 49 orange, 49 yellow, 49 red, all 32 green, and all 16 blue tickets ($49 + 49 + 49 + 32 + 16 = 195$).

Since all green and blue tickets would have been drawn from the box, the next ticket selected would have to be the 50th orange, yellow or red ticket.

Thus, the smallest number of tickets that must be drawn to ensure that at least 50 tickets of one colour have been selected is 196.

ANSWER: D