



Grade 7/8 Math Circles
February 25/26, 2014
Probability - Solutions

Exercises I

1. What is the sample space for rolling a fair six-sided die and flipping a fair coin?
 $\{ H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6 \}$
2. State whether the following pairs of events are Mutually-Exclusive, Independent or Dependent (**Note:** assume all are without replacement):
 - (a) Drawing a 7 from a deck of cards AND drawing a Jack next.
Dependent
 - (b) Drawing a 7 AND an 8 from a deck in one draw.
Mutually-Exclusive
 - (c) Getting a head on a coin-toss AND getting a tail on a different coin-toss.
Independent
3. Find the probability of the following events:
 - (a) Drawing a Spade from a deck of cards
 $\frac{4}{52} = \frac{1}{13}$
 - (b) Drawing a face card (J, Q, K, A) from a deck of cards.
 $\frac{16}{52} = \frac{4}{13}$
 - (c) Drawing a 6 from a deck after three 6's have been removed from the deck.
 $\frac{1}{49}$
 - (d) Drawing a 6 from a deck after all the face cards have been removed.
 $\frac{4}{36} = \frac{1}{9}$

Problem Set

1. Which definition of probability is being used here?

(a) Participating in a raffle.

Classical OR Experimental

(b) Being 90% sure you passed your math test.

Subjective

(c) Testing several products in a production line to see if they are defective.

Experimental

2. Find the missing probability that makes the set a Probability Distribution:

(a) $P(0) = \frac{1}{7}$, $P(1) = \frac{2}{7}$, $P(2) = X$, $P(3) = \frac{3}{7}$

$$\begin{aligned}1 &= \frac{1}{7} + \frac{2}{7} + X + \frac{3}{7} \\1 - \frac{1}{7} - \frac{2}{7} - \frac{3}{7} &= X \\ \frac{1}{7} &= X\end{aligned}$$

(b) $P(0) = X$, $P(1) = \frac{2}{5}$, $P(2) = \frac{3}{7}$

$$\begin{aligned}1 &= X + \frac{2}{5} + \frac{3}{7} \\1 - \frac{2}{5} - \frac{3}{7} &= X \\1 - \frac{14}{35} - \frac{15}{35} &= X \\ \frac{6}{35} &= X\end{aligned}$$

3. $P(A) = 0.6$ and $P(B) = 0.4$. If $P(A \cap B) = 0.25$ are:

(a) A and B be independent?

No. If A and B are independent, then $P(A \cap B) = P(A)P(B)$. But $P(A)P(B) = 0.24$ when it should be 0.25.

(b) A and B jointly exhaustive?

Yes. A and B are jointly exhaustive if $P(A) + P(B) = 1$.

4. When rolling two fair dice, what is the probability that the sum is:

(a) greater than 5?

$$\frac{26}{36} = \frac{13}{18}$$

(b) less than 2?

$$0$$

(c) equal to 8?

$$\frac{5}{36}$$

(d) less than or equal to 12?

$$1$$

5. A weighted coin is altered so the probability of it landing on a head for each flip is $\frac{5}{7}$. The trick coin is flipped 3 times. What is the probability of getting a tail on the first flip and heads on the next two flips?

$$\frac{2}{7} \times \frac{5}{7} \times \frac{5}{7} = \frac{50}{343}$$

6. A regular coin is flipped and then a card is randomly drawn from a standard deck of 52 cards.

(a) Determine the probability of flipping a head, then drawing a diamond.

$$\frac{1}{2} \times \frac{4}{52} = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26}$$

(b) Determine the probability of flipping a head, then drawing a diamond or a heart.

$$\frac{1}{2} \times \left(\frac{13}{52} + \frac{13}{52} \right) = \frac{1}{2} \times \frac{26}{52} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

7. In a coin toss, what is the probability that heads is flipped exactly two times out of three tosses? What if the coin is weighted so that the probability that heads occurs is 0.6?

$$\text{Normal: } \binom{3}{2} (0.5)^2 (0.5)^1 = 0.375 \text{ OR } 37.5\%$$

$$\text{Weighted: } \binom{3}{2} (0.6)^2 (0.4)^1 = 0.432 \text{ OR } 43.2\%$$

8. What is the probability of winning the Lotto 6/49? (**Note:** Lotto 6/49 is played such that there is 49 numbers and you must pick 6 without repeats.)

$$\frac{1}{\binom{49}{6}} = \frac{1}{\frac{49 \times 48 \times 47 \times 46 \times 45 \times 44}{6 \times 5 \times 4 \times 3 \times 2 \times 1}} = \frac{1}{13983816}$$

9. Mark has a bag that contains 3 black marbles, 6 gold marbles, 2 purple marbles, and 6 red marbles. Mark adds a number of white marbles to the bag and tells Susan that if she now draws a marble at random from the bag, the probability of it being black or gold is $\frac{3}{7}$. How many white marbles does Mark add to the bag?

First we had $P(\text{Black or Gold}) = \frac{9}{17}$. Now we have $P(\text{Black or Gold}) = \frac{3}{7} = \frac{9}{21}$. This means that 4 white marbles were added.

10. * In a deck of 52 cards, how many ways can you choose 5 cards having at least 2 kings?

$$\left(\binom{4}{2} \times \binom{48}{3}\right) + \left(\binom{4}{3} \times \binom{48}{2}\right) + \left(\binom{4}{4} \times \binom{48}{1}\right) = 103776 + 4512 + 48 = 108336$$

11. * In Canada, the probability that someone plays baseball and/or hockey is 0.79. The probability that someone plays just hockey is 0.6 and the probability that some plays baseball and hockey is 0.15. What is the probability that some plays only baseball?

Let event A be playing hockey and event B be playing baseball:

We will be using: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cup B) = 0.79, P(A \cap B) = 0.15, P(A) = 0.6$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.79 = 0.6 + P(B) - 0.15$$

$$0.79 = 0.45 + P(B)$$

$$0.34 = P(B)$$

12. * A credit card PIN of length 4 is formed by randomly selecting (with replacement) 4 digits from the set 0 - 9. Find the probability:

- (a) the PIN is even

$$P(A) = \frac{10 \times 10 \times 10 \times 5}{10 \times 10 \times 10 \times 10} = \frac{5000}{10000} = \frac{1}{2}$$

- (b) the PIN has only even digits

$$P(A) = \frac{5 \times 5 \times 5 \times 5}{10 \times 10 \times 10 \times 10} = \frac{625}{10000} = \frac{1}{16}$$

(c) ** the PIN contains at least one 1

Instead of looking at all the numbers with at least one 1, let's look at all the numbers without a 1 and use the indirect method:

There are 10^4 numbers total and 9^4 of them do not have a 1. Therefore $10^4 - 9^4$

numbers have at least one 1.

$$\frac{10^4 - 9^4}{10^4} = \frac{3439}{10000}$$

13. * The 10,000 tickets for a lottery are numbered 0000 to 9999. A four-digit winning number drawn is and a prize is paid on each ticket whose four-digit number is any arrangement of the number drawn. For example, if the winning number 0011 is drawn, the grand prize is split between the people who hold 0011, 0101, 0110, 1001, 1010 and 1100. What is the probability of winning with:

(a) 6446?

Using Permutations with Repetitions: $\frac{4!}{2!2!} = 6$ different permutations.

Since there are 10000 tickets, the probability is $\frac{6}{10000}$ or $\frac{3}{5000}$

(b) 7843?

Using Basic Permutations: $4! = 24$ different permutations.

Since there are 10000 tickets, the probability is $\frac{24}{10000}$ or $\frac{3}{1250}$

14. ** A student randomly guesses the correct answer in a multiple choice quiz of 10 question. If each question has 5 choices, what is the probability the student gets exactly 7 correct? What is the probability the students gets more than 7 correct?

Exactly 7: $\binom{10}{7}(0.2)^7(0.8)^3 = 0.000786$ OR 0.0786%

More than 7: $\binom{10}{8}(0.2)^8(0.8)^2 + \binom{10}{9}(0.2)^9(0.8)^1 + \binom{10}{10}(0.2)^{10}(0.8)^0$

15. ** A company owns 300 PS4 consoles. 8% of the consoles do not work. You randomly select 25 consoles.

Note: 0.08% of the council do not work which means 276 work and 24 do not work.

(a) What is the probability that all consoles are working?

$$P(0) = \frac{\binom{276}{25}}{\binom{300}{25}}$$

(b) What is the probability that 1 or 2 consoles are not working?

$$P(x = 1 \text{ or } 2) = \frac{\binom{24}{1} \binom{276}{24} + \binom{24}{2} \binom{276}{23}}{\binom{300}{25}}$$

(c) What is the probability that more than 3 consoles are not working?

$$P(x > 3) = 1 - (P(0) + P(1) + P(2) + P(3)) =$$
$$1 - \frac{\binom{24}{0} \binom{276}{25}}{\binom{300}{25}} - \frac{\binom{24}{1} \binom{276}{24}}{\binom{300}{25}} - \frac{\binom{24}{2} \binom{276}{23}}{\binom{300}{25}} - \frac{\binom{24}{3} \binom{276}{22}}{\binom{300}{25}}$$