Intermediate Math Circles
October 8, 2014
Geometry I: Angles

Over the next four weeks, we will look at several geometry topics. Some of the topics may be familiar to you while others may not. We will start with angle facts and some simple proofs.

Warming Up Let us try some riddles from www.bigriddles.com.

a) In a bouquet of flowers, all but two are roses, all but two are tulips, and all but two are daisies. How many flowers are in the bouquet?

b) You have two lengths of rope. Each rope has the property that if you light it on fire at one end, it will take exactly 60 minutes to burn to the other end. Note that the ropes will not burn at a consistent speed the entire time (for example, it's possible that the first 90 % of a rope will burn in 1 minute, and the last 10 % will take the additional 59 minutes to burn). Given these two ropes and a matchbook, can you find a way to measure out exactly 45 minutes?

c) You are walking down a path when you come to two doors. Opening one of the doors will lead you to a large pile of cash, while opening the other door will lead to a pride of hungry lions and lionesses. You don't know which door leads to which situation. In front of the doors are twin brothers who know which door leads to what. One of the brothers always lies, and the other always tells the truth. You don't know which brother is the liar and which is the truth-teller. You are allowed to ask one single question to one of the brothers (not both) to figure out which door to open. What question should you ask?

Getting Started

In geometry, there are certain terms that we just understand. For example, if you were asked to define an angle, you might struggle with appropriate words but could easily draw an angle. Try writing a definition below:

Let us discuss a short list of definitions that will be useful in our discussions.

Angle Related Definitions

1. A/an _____
   is any angle measuring between 0° and 90°.

2. A/an _____
   is an angle measuring 90°.

3. A/an _____
   is any angle measuring between 90° and 180°.

4. A/an _____
   is an angle measuring 180°.

5. Two angles whose sum is 180° are called

   _____.

6. Two angles whose sum is 90° are called

   _____.

7. A/an _____
   triangle has three sides of different length.

8. A/an _____
   triangle has two sides of equal length.

9. A/an _____
   triangle has three sides of equal length.

10. When two lines intersect, four angles are formed. The angles that are directly opposite to each other are called

   _____.

Prove: Opposite angles are equal.

Proof:
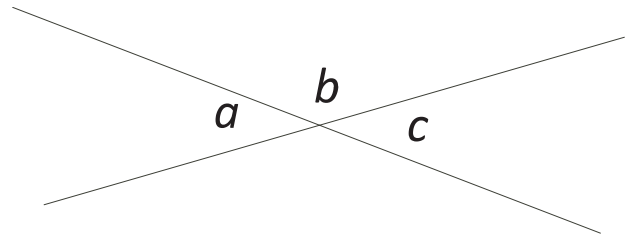
11. A/an _____ is any line that intersects two (or more) lines at different points.
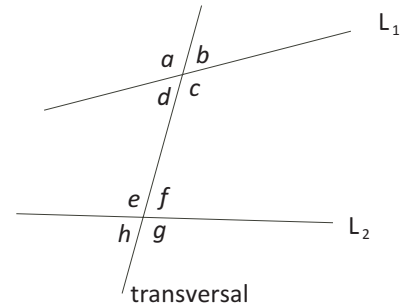
transversal

12. Two angles located on the same side of the transversal be-

tween the two lines are called _____.

13. Two non-adjacent angles located on opposite sides of the transversal be-

tween the two lines are called _____.

14. Two non-adjacent angles located on opposite sides of the transversal but not between the two

lines are called _____.

15. Two angles located at the same location relative to where the transversal intersects each line are

called _____.

16. A/an _____ is a logical statement which is assumed to be true. We just accept the truth of the statement.

Axiom:

If a transversal cuts two parallel lines, then the co-interior angles are supplementary. That is, the two co-interior angles add to 180°.
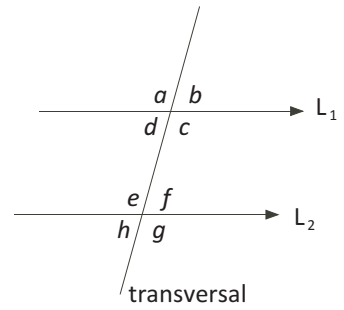
Another axiom that we could have started with is: "the angles in a triangle sum to 180°." As a result of starting with the first axiom, we will be able to prove the second axiom (it will not be an axiom for us).

Prove:

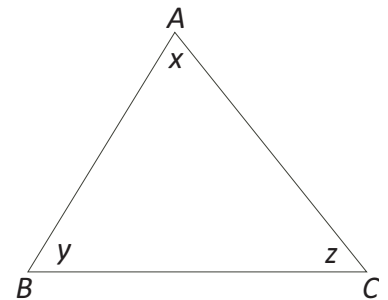If a transversal cuts two parallel lines, then the alternate angles are equal and the corresponding angles are equal.

Proof:
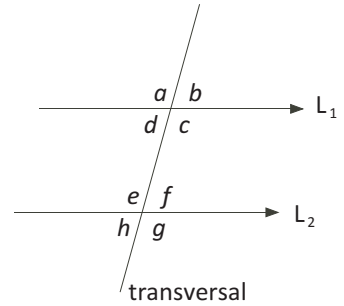
Prove:

In any triangle, the sum of the interior angles is 180°.

Proof:

Definition:

A/an _____ is the angle between one side of a triangle and the extension of an adjacent side.

Prove:

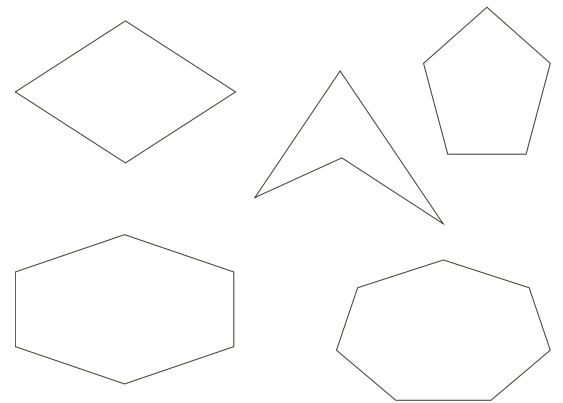An exterior angle of a triangle is equal to the sum of the opposite interior angles.

Proof:
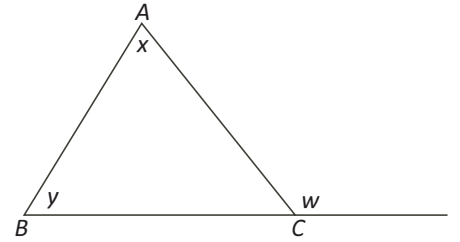
Definition:

A _____ is a closed plane figure having three or more sides. Triangles, rectangles, and octagons are all examples of polygons.

A _____ polygon is a polygon in which all sides are the same length and all interior angles are the same measure.

  a) Determine the sum of the interior angles in a quadrilateral.

  b) Determine the sum of the interior angles in a pentagon.

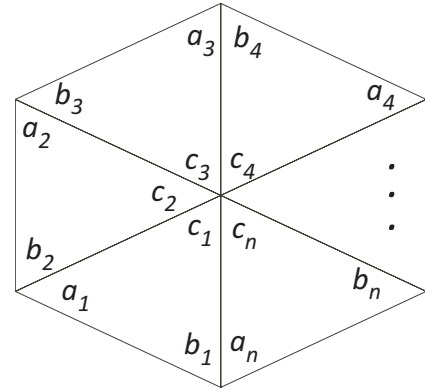 c) Determine the sum of the interior angles in a hexagon.

  d) Determine the sum of the interior angles in a heptagon.

Based on this pattern, what would be your conjecture for a formula for the sum of the interior angles in an n-gon?

Prove:

The sum of the interior angles in an n-sided polygon is $180° \times (n-2)$.
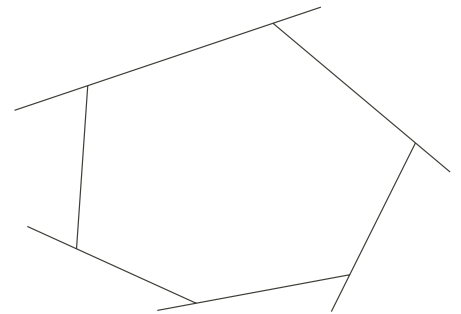
Proof:

Since each angle in a regular polygon is equal, then it follows that each angle in a regular n-sided

polygon is _____.

Prove:

The sum of the exterior angles in an n-sided polygon is $360°$.

Proof:

Prove:

If a triangle is isosceles, then the angles opposite the equal sides are equal.

Proof: